Question 1

\[ x = t^3 - 6t^2 + 9t. \]

a) Velocity \[ \frac{dx}{dt} = 3t^2 - 12t + 9. \]

Acceleration \[ \frac{dv}{dt} = 6t - 12. \]

b) at rest when velocity is zero. So use velocity formula

\[ \frac{3t^2 - 12t + 9}{6t - 12} = 0 \]

\[ t^2 - 4t + 3 = 0 \]

\[ t^2 - 4t + 3 = 0 \]

\[ \begin{bmatrix} (-3)(-1) = 3 \\ (-3)(1) = -3 \end{bmatrix} \]

\[ (t-3)(t-1) = 0 \]

\[ t-3 = 0 \rightarrow t = 3 \]

\[ t-1 = 0 \rightarrow t = 1 \]

Positions at \( t = 1 \), \( t = 3 \) relate to displacement so we use

\[ x = t^3 - 6t^2 + 9t \]

for \( t = 1 \)

\[ x = (1)^3 - (6\times1) + (9\times1) \]

\[ = 1 - 6 + 9 \]

\[ = 4 \text{ units} \]

For \( t = 3 \)

\[ x = (3)^3 - 6\times(3)^2 + (9\times3) \]

\[ = 27 - 54 + 27 \]

\[ = 0 \text{ units} \]
(c) \( t = 2 \)

\[
\text{velocity } = \frac{dx}{dt} = 3t^2 - 12t + 9
\]

\[
= [3 	imes (2)^2] - (12 	imes 2) + 9
\]

\[
= 12 - 24 + 9
\]

\[
= -3 \text{ m/s} = -3 \text{ m/s}
\]

Minus = particle reversing direction / returning to origin.

(d) \( \frac{dv}{dt} = 6t - 12 = 0 \).

\[ 6t - 12 = 0 \]

\[ 6t = 12 \]

\[ t = \frac{12}{6} = 2 \]

Acceleration is zero at 2 secs. - This is a minimum value as curve of a true quadratic \( 3t^2 - 12t + 9 \) is a parabola with a minimum point.

A velocity curve would be.

\[
\text{Gradient } = \text{zero } = \text{acceleration} \}
\]

\[
\text{+ minimum point}
\]
Question 2

Displacement = \( x = 3t^2 - t^3 \)

(i) Particle returns to origin when displacement = 0.
\[
3t^2 - t^3 = 0
\]
\[
t^2(3-t) = 0
\]
\[
t^2 = 0 \times 3 - t = 0 \Rightarrow t = 3 \text{ secs.}
\]

(ii) Speed = velocity \( \frac{dx}{dt} = 6t - 3t^2 \)

Returns to origin after 3 secs:

\[
speed = 6t - 3t^2
\]
\[
= (6 \times 3) - 3 \times (3)^2
\]
\[
= 18 - 27 = -9 \text{ units/sec}
\]

(iii) Particle at rest when \( \frac{dx}{dt} \) (velocity) = 0.

\[
6t - 3t^2 = 0
\]
\[
3(2 - t) = 0
\]
\[
3t = 0 \Rightarrow t = 0 \Rightarrow 0 \text{ secs.}
\]
\[
2 - t = 0 \Rightarrow t = 2 \text{ secs.}
\]

Position at times of rest: - (Use displacement formula).

For \( t = 0 \) \( x = 3t^2 - t^3 \)
\[
= (3 \times 0) - 0^3 = 0 \text{ units}
\]

For \( t = 2 \text{ sec} \) \( x = \left[ 3 \times (2)^2 \right] - (2)^3 = 12 - 8 = 4 \text{ units} \)
(iv) Acceleration = \( \frac{dv}{dt} = 6 - 6t \)

\[ 6 - 6t = 0 \]
\[ 6 = 6t \]
\[ t = \frac{6}{6} = 1 \text{ sec} \]

Speed:
\[ \frac{dx}{dt} = 6t - 3t^2 \]
\[ = \left( 6 \times 1 \right) - \left[ 3 \times 1^2 \right] \]
\[ = 6 - 3 \]
\[ = 3 \text{ units} \text{ s}^{-1} \]

When speed =

When acceleration is zero on a velocity/time graph, the gradient of the tangent to the curve is zero.

As we have a \(-ve\) coefficient to \( 6t - 3t^2 \)

\(-3t^2\) Quadratic part.

we have a maximum velocity; sketch.

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Question 3.

\[ s = t(t-4)^2 \]

\[ (t-4)(t-4) = t^2 - 4t - 4t + 16 \]
\[ t^2 - 8t + 16. \]

\[ t(t^2 - 8t + 16). \]
\[ t^3 - 8t^2 + 16t. \]

Velocity \[ \frac{ds}{dt} = 3t^2 - 16t + 16 \]

b) Particle at rest when velocity \[ \frac{ds}{dt} = 0. \]

\[ 3t^2 - 16t + 16 = 0 \]

\[ a = 3, \quad b = -16, \quad c = 16. \]

\[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{16 \pm \sqrt{(-16)^2 - 4 \times 3 \times 16}}{2a}. \]

\[ \frac{16 \pm \sqrt{256 - 192}}{6} \]

\[ \frac{16 \pm \sqrt{64}}{6} = \frac{16 \pm 4}{6} \]

\[ \frac{16 + 4}{6} \quad \text{and} \quad \frac{16 - 4}{6} \]

\[ \frac{16 + 8}{6} = \frac{24}{6} = 4 \text{ secs.} \]

\[ \frac{16 - 8}{6} = \frac{8}{6} = \frac{4}{3} \text{ secs.} \]

Time at rest 4 secs and \( \frac{4}{3} \) secs.
Question 3 (cont).

c) Particle has zero acceleration: when:

(i) First find formula for acceleration:

\[ \frac{ds}{dt} = 3t^2 - 16t + 16 \]

\[ \text{acceleration } = \frac{dv}{dt} = 6t - 16 \]

\[ 6t - 16 = 0 \]

\[ 6t = 16 \]

\[ t = \frac{16}{6} = \frac{8}{3} = 2.66 \]

\[ = 2.7 \text{ secs.} \]

d) Related to displacement when acceleration is zero:

\[ s = t^3 - 8t^2 + 16t \]

\[ = \left( \frac{8}{3} \right)^3 - \left[ 8 \times \left( \frac{8}{3} \right)^2 \right] + \left[ 16 \times \frac{8}{3} \right] \]

\[ = 4.56 \text{ m from origin. (2.D.P.)} \]

What is it doing? Need to find velocity and hence direction. Also has zero acceleration at velocity as coefficient in velocity derivative is positive ie \( 3t^2 \).

\[ \frac{ds}{dt} = 3t^2 - 16t + 16 \]

\[ 3 \times (2.7)^2 - (16 \times 2.7) + 16 \]

\[ = -5.33 \text{ m/s} \] minus means returning to its origin.
Velocity-Time Graph of Particle

\[ 3t^2 - 16t + 16 \]

Zero Gradient / Zero Acceleration
10 min Stationary Point
Q15: \( s = t^3 - 6t + 3 \)

a) \( \text{Velocity} = \frac{ds}{dt} = 3t^2 - 6 \text{ m/s} \)

b) \( \text{Acceleration} = \frac{dv}{dt} = 6t \)

After 5 secs: \( 6 \times 5 = 30 \text{ m/s}^2 \)

May 2008 4H Paper, Q19

\( s = t^2 - 6t + 10 \)

\( \frac{ds}{dt} = \text{Velocity} = 2t - 6 \)

b) At \( T = 5 = (2 \times 5) - 6 \)

\( = 10 - 6 \)

\( = 4 \text{ m/s} \)

c) \( \text{Acceleration} = \frac{dv}{dt} = 2 \text{ m/s}^2 \)

May June 2006 4H Paper Q18

\( x = \frac{20}{t} \rightarrow \text{displacement} = x = \frac{20}{t} = 20t^{-1} \)

\( \text{Velocity} = \frac{dx}{dt} = 4 - 20t^{-2} \text{ m/s} \)

\( \text{Acceleration} = \frac{dv}{dt} = 40t^{-3} \text{ m/s}^2 \) or \( 40 \text{ m/s}^2 \)

\( \frac{t^3}{t^3} \)

\[ s = t^3 + 4t^2 - 5t \]

\( a) \quad \text{velocity } = \frac{ds}{dt} = 3t^2 + 8t - 5 \)

\( b) \quad \text{acceleration } = \frac{dv}{dt} = 6t + 8 \)

after 2 secs = \((6 \times 2) + 8\)

\[ 12 + 8 = 20 \text{ m/s}^2 \]


\[ s = 2t^3 - 12t^2 + 7t \]

\( a) \quad \text{velocity } = \frac{ds}{dt} = 6t^2 - 24t + 7 \)

\[ \text{acceleration } = \frac{dv}{dt} = 12t - 24 = 0 \]

\( b) \quad 12t - 24 = 0 \)

\[ 12t = 24 \]

\[ t = \frac{24}{12} = 2 \text{ secs.} \]
Nov 2009 4H Paper B19

\[ s = t^3 - 5t^2 + 8 \]

(a) \[
velocity = \frac{ds}{dt} = 3t^2 - 10t
\]

(b) \[
acceleration = \frac{dv}{dt} = 6t - 10
\]

\[ 20 = 6t - 10 \]
\[ 20 + 10 = t = \frac{30}{6} = 5 \text{ secs.} \]

May 2013 / 3H Paper, A19

(a) \[
velocity = \frac{ds}{dt} = 18t - 3t^2 \text{ m/s}
\]

(b) \[
acceleration = 0 = \frac{dv}{dt} = 18 - 6t.
\]

\[ 18 - 6t = 0 \]
\[ -6t = -18 \]
\[ t = \frac{-18}{-6} = \frac{-9}{-3} = 3 \text{ secs.} \]