May
June 2007 3H Paper 817

\[ y = \frac{x^2 + 16}{x} \]

\[ y = x^2 + 16x^{-1} \]

\[ \frac{dy}{dx} = 2x + (-16x^{-2}) \]

\[ 2x - 16x^{-2} \text{ OR } 2x - \frac{16}{x^2} \]

**Turning point when** \( 2x - 16 = 0 \).

\[ 2x = \frac{16}{x^2} \]

\[ x^2(2x) = 16 \]

\[ 2x^3 = 16 \]

\[ x^3 = \frac{16}{2} = 8 \]

\[ x = \sqrt[3]{8} \]

\[ x = 2 \]

\[ y = \frac{x^2 + 16}{x} \]

\[ \frac{x^2}{x} + \frac{16}{x} \rightarrow \frac{(2^2)}{2} + \frac{16}{2} \rightarrow 4 + 8 \]

\[ y = 12 \]

*(Ans: Turning point 2, 12)*
a) $A = 36x - x^2$

Perimeter = $2x + 2y = 72$
$x + y = 36$

$y = 36 - x$

$A = L \times W$
$= (36-x) \times x$
$= 36x - x^2$

b) $\frac{dA}{dx} = 36 - 2x$

c) Max value of $A$ when gradient is zero.
curve is inverted parabola since we have a NEGATIVE quadratic component

$36 - 2x = 0$
$-2x = -36$
$x = \frac{-36}{-2} = 18$

Curve is max also since $\frac{d^2A}{dx^2} = -2$ which is < 0.
Jan 2014 3HR Paper Q18.

Total amount of fencing = 120 metres.

(i) To cover all perimeter:

\[2y + 3x\]

\[2y + 3x = 120,\]

\[y + 1.5x = 60,\]

\[y = 60 - 1.5x.\]

(ii) Area of rectangle = \(\pi \times y\)

\[= \pi \times (60 - 1.5x),\]

\[= 60\pi - 1.5\pi x^2.\]

b) \(\frac{dA}{dx} = 60 - 3x.\)

c) Maximum = \(60 - 3x = 0\) → \(-3x = -60\)

\[x = \frac{-60}{-3} = 20.\text{m}.\]

Find \(x\).

If \(x = 20\) then

\[2y + 3x = 120,\]

\[2y + (3\times 20) = 120,\]

\[2y + 60 = 120.\]

\[2y = 120 - 60 = 60,\]

\[y = 60/2 = 30\text{m}.\]

Area = \(L \times W = 30 \times 20 = 600\text{m}^2.\)
May 2014, 3H Paper, Q14

a (i) \(12x + 10y = 180\)
\[6x + 5y = 90.\]
\[5y = 90 - 6x \implies y = \frac{90 - 6x}{5}\]
\[y = 18 - 1.2x\]

(ii) \(\text{Area} = 144x - 9.6x^2\)
\[A = l \times b\]
\[= 4x \times 2y\]
\[= 4x \times 2(18 - 1.2x)\]
\[= 4x(36 - 2.4x)\]
\[= 144x - 9.6x^2\]

b) \(\frac{dA}{dx} = 144 - 19.2x \implies\)

C) Max when \(144 - 19.2x = 0\).
\[-19.2x = -144\]
\[x = \frac{-144}{-19.2} = 7.5\]
\[A = 144x - 9.6x^2\]
\[(144 \times 7.5) - 9.6 \times (7.5)^2\]
\[= 1080 - 540 = 540 \text{m}^2\]
a) \[ 4y + 8x = 112 \]
\[ 4y = 112 - 8x \]
\[ y = 28 - 2x. \]

\[ V = L \times B \times H \]
\[ = (28 - 2x) \times x \times x \]
\[ = (28 - 2x) x^2 \]
\[ = x^2 (28 - 2x). \]
\[ V = 28x^2 - 2x^3 \]

b) \[ \frac{dv}{dt} = 56x - 6x^2 \]

\[ \frac{56x - 6x^2}{dt} = 0 \]
\[ 28 - 3x = 0 \]
\[ 2x (28 - 3x) = 0 \]
\[ 2x = 0 \]
\[ x = \frac{28}{3} = 9.33 \]

\[ V = L \times 28x^2 - 2x^3 \]
\[ \left[ 28x \left( \frac{28}{3} \right)^2 \right] - \left[ 2x \left( \frac{28}{3} \right)^3 \right] \]
\[ = 813.037 \]
\[ = 813 \text{ cm}^3. \]
Nov 2006 3H Paper Q14

(a) \[ y = 5000x - 625x^2 \]

\[ \frac{dy}{dx} = 5000 - 1250x. \]

(b) T.P is at \[ 5000 - 1250x = 0 \]

\[ -1250x = -5000 \]

\[ x = \frac{-5000}{-1250} \]

\[ x = 4 \]

\[ y = (5000 	imes 4) - 625 	imes (4^2) = 12. \]

(16) (coordinates of T.P = (4,16)).

(c) \[ -x^2 = \text{minu} \text{t co-efficient.} = \text{maximum.} \]

(i) + (ii).

(d) (i)

\[ \text{Profit} \]

\[ \text{Price} \]

\[ \£4 \]

\[ \text{A} \]

\[ \text{B} \]

\[ \£4 \text{ Asking price} \]

(ii) It was the max point of profit as illustrated on the sketch graph for part c.

Fencing = 28 m

a) Area = \( y = 28x - 2x^2 \)

\[ A = L \times W \]
\[ = (28 - 2x) \times x \Rightarrow x(28 - 2x) \Rightarrow 28x - 2x^2 \]

b) (i) \( \frac{dy}{dx} = 28 - 4x \)

(ii) \( y = \) maximum at \( 28 - 4x = 0 \)

\[ 28 - 4x = 0 \]
\[ -4x = -28 \]
\[ x = \frac{-28}{4} \]
\[ = -7 \]

(iii) \( \text{Max as } y = 28x - 2x^2 \) quadratic part has -ve coefficient

c) Area = \( y = 28x - 2x^2 \)

\[ = (28 \times 7) - (2 \times (\frac{1}{4})^2) \]
\[ = 196 - (2 \times 1) \]
\[ = 194 \text{ m}^2 \]
\[ A = 2x^2 - 18x + 80. \]

\[ 80 - (10x - x^2) - (8x - x^2). \]
\[ = 80 - 1(10x - x^2) - 1(8x - x^2). \]
\[ = 80 - 16x + x^2 - 8x + x^2 \]
\[ = 80 - 18x + 2x^2. \]
\[ A = 2x^2 - 18x + 80. \]

b) \[ A = 2x^2 - 18x + 80 \]

(i) \[ \frac{dA}{dx} = 4x - 18 \]

(ii) \[ A \text{ is a minimum when } \frac{dA}{dx} = 0. \]
\[ 2x^2 - 18x + 80 = 0. \]
\[ x^2 - 9x + 40 = 0. \]
\[ 4x - 18 = 0 \]
\[ 4x = 18 \]
\[ x = \frac{18}{4} = \frac{9}{2} = 4.5 \]
(iii) \[ 2x^2 - 18x + 80 \]

The coefficient of \( x \) parabola has a minimum value.

Or \[ \frac{d^2A}{dx^2} \] of \( 4x - 18 \) is \( 4 \)

\[ 4 > 0 \] \( \Rightarrow \) parabola is 'U' shaped with minimum.