20. $A$ is the point with coordinates $(2, 3)$.

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}.$$  

Find the coordinates of $B$. 

OPQR is a rectangle.

D is the point on OP such that \( OD = \frac{1}{3} \) \( OP \).

E is the point on OQ such that \( OE = \frac{2}{3} \) \( OQ \).

PQF is the straight line such that \( QF = \frac{1}{3} \) \( PQ \).

\( \overrightarrow{OD} = a \quad \overrightarrow{OR} = 3b \)

(a) Find, in terms of \( a \) and \( b \),

(i) \( \overrightarrow{OQ} \)  (ii) \( \overrightarrow{OE} \)  (iii) \( \overrightarrow{DE} \)

(b) Use a vector method to prove that DEF is a straight line.
$OABC$ is a parallelogram.
$BCD$ is a straight line.
$BD = 3BC$.
$M$ is the midpoint of $OC$.
$\overrightarrow{OA} = x \quad \overrightarrow{AB} = y$

(a) Find, in terms of $x$ and $y$,

(i) $\overrightarrow{AM}$
(ii) $\overrightarrow{OD}$

(b) Use your answers to (a)(i) and (ii) to write down two different geometric facts about the lines $AM$ and $OD$. 
21

\[ \text{Diagram NOT accurately drawn} \]

\[ \overrightarrow{AB} = 12a \]
\[ \overrightarrow{AD} = 3b \]
\[ \overrightarrow{DC} = 18a \]

\[ E \text{ is the point on the diagonal } DB \text{ such that } DE = \frac{1}{3} DB. \]

(a) Find, in terms of \( a \) and \( b \),

(i) \( \overrightarrow{DB} \)

(ii) \( \overrightarrow{DE} \)

(iii) \( \overrightarrow{AE} \)

(b) Show by a vector method that \( BC \) is parallel to \( AE \).
21

$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

$P$ is the point on $AB$ such that $AP = \frac{1}{4} AB$.

$Q$ is the point on $OC$ such that $OQ = \frac{2}{3} OC$.

Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, $\overrightarrow{PQ}$.

Give your answer in its simplest form.
Vectors IGCSE Higher Tier Exam Questions

Jan 2015 4H Paper

19 \[ \mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -7 \\ 0 \end{pmatrix} \]

(a) Write, as a column vector, \(2\mathbf{a}\)

(b) Write, as a column vector, \(3\mathbf{b} - \mathbf{c}\)

(c) Work out the magnitude of \(\mathbf{a}\)
   Give your answer as a surd.

Jan 2015 4HR Paper

15 Here is the parallelogram \(ABCD\).

\[
\overrightarrow{AD} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}
\]
(a) Find the magnitude of $\overrightarrow{AD}$.

Give your answer correct to 3 significant figures.

The point $A$ has coordinates $(4, 2)$

(b) Work out the coordinates of the point $C$.

The diagonals of the parallelogram $ABCD$ cross at the point $E$.

(c) Find as a column vector, $\overrightarrow{OE}$.

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In triangle $OPQ$, $\overrightarrow{OP} = 6a$ and $\overrightarrow{OQ} = 6b$

$X$ is the midpoint of $PQ$.

(a) Find, in terms of $a$ and $b$, the vector $\overrightarrow{OX}$

Give your answer in its simplest form.

$Y$ is the point on $OX$ such that $OY : YX = 2 : 1$

(b) Find, in terms of $a$ and $b$, the vector $\overrightarrow{QY}$

Give your answer in its simplest form.
$PQ$ is a triangle.
The midpoint of $PQ$ is $W$.
$X$ is the point on $QR$ such that $QX : XR = 2 : 1$
$PRY$ is a straight line.

$\overrightarrow{PW} = a, \overrightarrow{PR} = b$

(a) Find, in terms of $a$ and $b$,

(i) $\overrightarrow{QR}$

(ii) $\overrightarrow{QX}$

(iii) $\overrightarrow{WX}$
17 $ABCD$ is a parallelogram.

$\overrightarrow{BC} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad \overrightarrow{DC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Find $\overrightarrow{BD}$ as a column vector.

Diagram NOT accurately drawn

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Jan 2016 3H Paper

23

$OAB$ is a triangle.

$P$ is the point on $OA$ such that $OP : PA = 2 : 1$

$C$ is the point such that $B$ is the midpoint of $OC$.

$M$ is the midpoint of $AB$.

$\overrightarrow{OA} = 6a \quad \overrightarrow{OB} = 4b$

Show that $PMC$ is a straight line.
19 The diagram shows a grid of equally spaced parallel lines. The point $P$ and the vectors $\mathbf{a}$ and $\mathbf{b}$ are shown on the grid.

\[
\overrightarrow{PQ} = 3\mathbf{a} + 4\mathbf{b}
\]

(a) On the grid, mark the vector $\overrightarrow{PQ}$

\[
\overrightarrow{PR} = -4\mathbf{a} + 2\mathbf{b}
\]

(b) On the grid, mark the vector $\overrightarrow{PR}$

(c) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector $\overrightarrow{QR}$

The point $M$ lies on $PR$ such that $PM = \frac{2}{3} PR$

The point $N$ lies on $PQ$ such that $PN = \frac{1}{3} PQ$

(d) Show that $\overrightarrow{MN} = k\mathbf{a}$ where $k$ is a constant. State the value of $k$. 

OMN is a triangle.

P is the point on OM such that \( OP = \frac{1}{4} OM \)

P is the point on OM such that \( OP = \frac{1}{4} OM \)

Q is the midpoint of ON
R is the midpoint of PN

\[ \overrightarrow{OP} = \mathbf{p} \quad \overrightarrow{OQ} = \mathbf{q} \]

(a) Find, in terms of \( \mathbf{p} \) and \( \mathbf{q} \),
   (i) \( \overrightarrow{MN} \)
   (ii) \( \overrightarrow{PR} \)

(b) Use a vector method to prove that QR is parallel to OP
\( \overrightarrow{AB} \) is parallel to \( \overrightarrow{DC} \)
\( \overrightarrow{DC} = 2\overrightarrow{AB} \)
\( M \) is the midpoint of \( BC \)
\( \overrightarrow{AD} = 2\mathbf{b} \)
\( \overrightarrow{AB} = 4\mathbf{a} \)

(a) Find \( \overrightarrow{BM} \) in terms of \( \mathbf{a} \) and \( \mathbf{b} \).
   Give your answer in its simplest form.

\( N \) is the point such that \( DCN \) is a straight line and \( DC : CN = 2 : 1 \)
(b) Show that \( AMN \) is a straight line.
21. \[ \text{Diagram NOT accurately drawn} \]

\[ \overrightarrow{PQRS} \text{ is a trapezium with } \overrightarrow{PQ} \text{ parallel to } \overrightarrow{SR}. \]
\[ \overrightarrow{SR} = \mathbf{a} \quad \overrightarrow{PQ} = 3\mathbf{a} \quad \overrightarrow{PS} = \mathbf{b} \]

\( T \) is the point on \( SQ \) such that \( ST = \frac{1}{4}SQ \).

(a) Find, in terms of \( \mathbf{a} \) and \( \mathbf{b} \),

(i) \( \overrightarrow{PR} \)

(ii) \( \overrightarrow{SQ} \)

(iii) \( \overrightarrow{PT} \)

(b) \( \overrightarrow{PT} = k \overrightarrow{PR} \) where \( k \) is a fraction.

(i) What does this result tell you about the points \( P, T \) and \( R? \)

(ii) Find the value of \( k \).
21.

In the diagram $\overrightarrow{OA} = a$ and $\overrightarrow{OC} = c$.

(a) Find $\overrightarrow{CA}$ in terms of $a$ and $c$.

(b) The point $B$ is such that $\overrightarrow{AB} = \frac{1}{2} c$.

Give the mathematical name for the quadrilateral $OABC$.

(c) The point $P$ is such that $\overrightarrow{OP} = a + kc$, where $k \geq 0$

State the two conditions relating to $a + kc$ that must be true for $OAPC$ to be a rhombus.
18. The diagram shows a parallelogram, $ABCD$.
   $M$ is the midpoint of $BC$.
   $N$ is the midpoint of $AD$.

\[
\overrightarrow{AB} = x \\
\overrightarrow{AD} = y
\]

Find, in terms of $x$ and/or $y$, the vectors

(a) $\overrightarrow{MN}$

(b) $\overrightarrow{AC}$

$P$ is the point such that $\overrightarrow{CP} = y - \frac{1}{2} x$

(c) Find, in terms of $x$ and/or $y$, the vector $\overrightarrow{PA}$
   Simplify your answer as much as possible.
19. The diagram shows a trapezium $ABCD$.

\[ \overrightarrow{BC} = 2\overrightarrow{AD}, \]
\[ \overrightarrow{AB} = x. \quad \overrightarrow{AD} = y. \]

(a) Find, in terms of $x$ and $y$,

(i) $\overrightarrow{AC}$

(ii) $\overrightarrow{DC}$

(b) The point $E$ is such that $\overrightarrow{AE} = x + y$.
Use your answer to part (a)(ii) to explain why $AECD$ is a parallelogram.
21. \( PQRSTU \) is a regular hexagon, centre \( O \).
The hexagon is made from six equilateral triangles of side 2.5 cm.

\[
\overrightarrow{TU} = \mathbf{a}, \quad \overrightarrow{UP} = \mathbf{b}.
\]

(a) Find, in terms of \( \mathbf{a} \) and/or \( \mathbf{b} \), the vectors

(i) \( \overrightarrow{TP} \)

(ii) \( \overrightarrow{PO} \)

(iii) \( \overrightarrow{UO} \)

(b) Find the modulus (magnitude) of \( \overrightarrow{UR} \).
16. \( PQR \) is a triangle.

\( E \) is the point on \( PR \) such that \( PR = 3PE \).
\( F \) is the point on \( QR \) such that \( QR = 3QF \).

\[ \overrightarrow{PQ} = \mathbf{a}, \quad \overrightarrow{PE} = \mathbf{b}. \]

\[ \overrightarrow{PQ} = \mathbf{a}, \quad \overrightarrow{PE} = \mathbf{b}. \]

(a) Find, in terms of \( \mathbf{a} \) and \( \mathbf{b} \),

(i) \( \overrightarrow{PR} \)

(ii) \( \overrightarrow{QR} \)

(iii) \( \overrightarrow{PF} \)

(b) Show that \( \overrightarrow{EF} = k \overrightarrow{PQ} \) where \( k \) is an integer.
14. $OABC$ is a parallelogram.

\[
\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.
\]

(a) Find the vector $\overrightarrow{OB}$ as a column vector.

(b) Find, in terms of $k$, the vectors

(i) $\overrightarrow{OX}$,

(ii) $\overrightarrow{AX}$,

(iii) $\overrightarrow{XC}$.

(c) Find the value of $k$ for which $\overrightarrow{AX} = \overrightarrow{XC}$.

(d) Use your answer to part (c) to show that the diagonals of the parallelogram $OABC$ bisect one another.
16. $PQR$ is a triangle.
   $M$ and $N$ are the midpoints of $PQ$ and $PR$ respectively.

\[ \overrightarrow{PM} = \mathbf{a} \quad \overrightarrow{PN} = \mathbf{b}. \]

(a) Find, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{MN}$

(ii) $\overrightarrow{PO}$

(iii) $\overrightarrow{QR}$

(b) Use your answers to (a)(i) and (iii) to write down two geometrical facts about the lines $MN$ and $QR$. 

22.

\[ PQRS \text{ is a parallelogram.} \]

\( X \) is the midpoint of \( QR \) and \( Y \) is the midpoint of \( SR \).

\( \overrightarrow{PQ} = \mathbf{a} \) and \( \overrightarrow{PS} = \mathbf{b} \).

(a) Write down, in terms of \( \mathbf{a} \) and \( \mathbf{b} \), expressions for

(i) \( \overrightarrow{PX} \)

(ii) \( \overrightarrow{PY} \)

(iii) \( \overrightarrow{QS} \)

(b) Use a vector method to show that \( XY \) is parallel to \( QS \) and that \( XY = \frac{1}{2} QS \).