Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – there may be more space than you need.
- Calculators may be used.
- You must NOT write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
International GCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

Pythagoras’ Theorem
\[ a^2 + b^2 = c^2 \]

Volume of cone = \( \frac{1}{3} \pi r^2 h \)
Curved surface area of cone = \( \pi rl \)
Volume of sphere = \( \frac{4}{3} \pi r^3 \)
Surface area of sphere = \( 4\pi r^2 \)

\begin{align*}
\text{adj} &= \text{hyp} \times \cos \theta \\
\text{opp} &= \text{hyp} \times \sin \theta \\
\text{opp} &= \text{adj} \times \tan \theta
\end{align*}

\text{In any triangle } \triangle ABC

\begin{align*}
\text{Sine rule: } \quad \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
\text{Cosine rule: } \quad a^2 &= b^2 + c^2 - 2bc \cos A
\end{align*}

\text{Area of triangle} = \frac{1}{2} ab \sin C

\text{Volume of prism} = \text{area of cross section} \times \text{length}

\text{Circumference of circle} = 2\pi r
\text{Area of circle} = \pi r^2

\text{Volume of cylinder} = \pi r^2 h
\text{Curved surface area of cylinder} = 2\pi rh

\text{Area of a trapezium} = \frac{1}{2}(a + b)h

\text{The Quadratic Equation}
The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1  There are 20 students in a class.
   12 of the students are girls.

   Find the ratio of the number of girls to the number of boys.
   Give your ratio in the form $n : 1$

   ............................................. : 1

   (Total for Question 1 is 2 marks)

2  Describe fully the single transformation which maps triangle $P$ onto triangle $Q$.

   ...........................................................................................................
   ...........................................................................................................

   (Total for Question 2 is 2 marks)

Do NOT write in this space.
3. The table shows information about the number of letters in the first name of each of 50 people.

<table>
<thead>
<tr>
<th>Number of letters</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

(i) Work out the mean number of letters in the first names of the 50 people.

(ii) One more person joins the 50 people. The mean number of letters in the first names of the 51 people is less than the mean number of letters in the first names of the 50 people.

Write down the greatest number of letters in the first name of the person who joins the group.

(Total for Question 3 is 4 marks)

Do NOT write in this space.
4 The diagram shows three points, \(A\), \(B\) and \(P\), on a centimetre grid.

![Diagram showing points A, B, and P on a grid]

The point \(A\) has coordinates \((4, 5)\) and the point \(B\) has coordinates \((2, -1)\).

(a) Find the coordinates of the midpoint of \(AB\).

\((\text{Answer})\)  

\((2)\)

\(AB\) is a diameter of a circle.  
\(P\) is the point \((7, 6)\)  
\(C\) is the point on the circle such that \(PA = PC\).

(b) On the diagram, mark with a cross \((\times)\) the point \(C\).  
Label your point \(C\).

\((2)\)

(Total for Question 4 is 4 marks)

Do NOT write in this space.
5 A shop, Furniture 4U, had a sale.

(a) In the sale, normal prices were reduced by 15%.

(i) The normal price of a table was $280
Work out the sale price of the table.

$..............................

(ii) The normal price of a chair was reduced in the sale by $24
Work out the normal price of the chair.

$..............................

(b) Ruth, Suha and Yasmin went to the sale.
The amounts of money spent by Ruth, Suha and Yasmin were in the ratios 2 : 3 : 7
Ruth and Suha spent a total of $320 in the sale.

Work out the amount of money Yasmin spent in the sale.

$..............................

(Total for Question 5 is 9 marks)
The diagram shows a pentagon $ABCDE$. DC is parallel to AB.

The size of an exterior angle at A is $67^\circ$
The size of an exterior angle at B is $112^\circ$
The size of an exterior angle at C is $x^\circ$
The size of an exterior angle at D is $74^\circ$
The size of an exterior angle at E is $y^\circ$

(a) (i) Work out the value of $x$.

(ii) Work out the value of $y$.

(b) Work out the sum of the interior angles of the pentagon $ABCDE$. 

(Total for Question 6 is 6 marks)
7 (i) Solve the inequalities \( 3 \leq x + 4 < 7 \)

(ii) \( n \) is an integer.

Write down all the values of \( n \) which satisfy \( 3 \leq n + 4 < 7 \)

(Total for Question 7 is 4 marks)

8

Calculate the value of \( x \).
Give your answer correct to 3 significant figures.

\( x = \ldots \)

(Total for Question 8 is 3 marks)
In the isosceles triangle $ABC$,  
$AB = AC$  
angle $B = (3x + 32)^\circ$  
angle $C = (87 - 2x)^\circ$  

Work out the value of $x$.  
Show clear algebraic working.

\[ x = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]  

(Total for Question 9 is 4 marks)
The grouped frequency table gives information about the weights of 180 airmail letters.

<table>
<thead>
<tr>
<th>Weight (w grams)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; $w \leq 20$</td>
<td>15</td>
</tr>
<tr>
<td>20 &lt; $w \leq 40$</td>
<td>25</td>
</tr>
<tr>
<td>40 &lt; $w \leq 60$</td>
<td>47</td>
</tr>
<tr>
<td>60 &lt; $w \leq 80$</td>
<td>70</td>
</tr>
<tr>
<td>80 &lt; $w \leq 100$</td>
<td>18</td>
</tr>
<tr>
<td>100 &lt; $w \leq 120$</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Weight (w grams)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; $w \leq 20$</td>
<td></td>
</tr>
<tr>
<td>0 &lt; $w \leq 40$</td>
<td></td>
</tr>
<tr>
<td>0 &lt; $w \leq 60$</td>
<td></td>
</tr>
<tr>
<td>0 &lt; $w \leq 80$</td>
<td></td>
</tr>
<tr>
<td>0 &lt; $w \leq 100$</td>
<td></td>
</tr>
<tr>
<td>0 &lt; $w \leq 120$</td>
<td></td>
</tr>
</tbody>
</table>
(b) On the grid, draw a cumulative frequency graph for your table.

(c) Find an estimate for the upper quartile of the weights of the 180 letters.

.................................. grams

(Total for Question 10 is 5 marks)

11  \[ A = 2^3 \times 3^3 \times 5^4 \]

\[ B = 3^5 \times 5 \times 7^3 \]

Find the Highest Common Factor (HCF) of \( A \) and \( B \).

..................................

(Total for Question 11 is 2 marks)
A and B are points on a circle, centre O. 
PA is the tangent to the circle at A. 
OBP is a straight line. 
Angle $APO = 26^\circ$ 
Calculate the size of angle $ABP$. 

(Total for Question 12 is 3 marks)
13 (a) Solve the simultaneous equations

\[
\begin{align*}
5x + 3y &= 9 \\
7x - 2y &= 25
\end{align*}
\]

Show clear algebraic working.

\[
x = \ldots............
\]

\[
y = \ldots............
\]  

(4)

(b) \(P\) is the point of intersection of the lines with equations \(5x + 3y = 9\) and \(7x - 2y = 25\)

Write down the coordinates of \(P\).

\((\ldots......, \ldots......)\)  

(1)

(Total for Question 13 is 5 marks)

14 Jomo invested an amount of money at 4% per annum compound interest.
At the end of 2 years, the value of his investment was £3380

How much of the £3380 was interest?

£ \ldots.............

(Total for Question 14 is 4 marks)
$D, E, F, G$ and $H$ are points on a circle. 
Angle $EGH = 67^\circ$

(a) Find the size of angle $EFH$.

(b) (i) Find the size of angle $EDH$.

(ii) Give a reason for your answer.

(Total for Question 15 is 3 marks)
16

Calculate the value of $x$.
Give your answer correct to 3 significant figures.

$$x = \ldots$$

(Total for Question 16 is 3 marks)

17 $y$ is directly proportional to $x^3$
When $x = 10$, $y = 250$

(a) Find a formula for $y$ in terms of $x$.

..............................................................

(b) Calculate the value of $x$ when $y = 54$

$$x = \ldots$$

(Total for Question 17 is 5 marks)
Here are ten counters. Each counter has a number on it.

Fern puts the ten counters in a bag. She takes at random a counter from the bag.

(a) Find the probability that the number on the counter is 3 or 4

Fern puts the counter back into the bag. Then Rajan takes at random one of the ten counters from the bag. He does not put the counter back into the bag. He then takes at random a second counter from the bag.

(b) Calculate the probability that 3 is the number on each of the two counters he takes.
(c) Calculate the probability that the sum of the numbers on the two counters he takes is an odd number.

19

The diagram shows a shape made from a solid cylinder and a solid hemisphere. The cylinder has a radius of 3.4 cm and a length of 8.3 cm. The hemisphere has a radius of 3.4 cm.

Calculate the total surface area of the solid shape. Give your answer correct to 3 significant figures.
20 (a) Complete the table of values for \( y = x^2 + \frac{2}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y  )</td>
<td>20.01</td>
<td>10.04</td>
<td>3</td>
<td>3.58</td>
<td>5</td>
<td>9.67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) On the grid, draw the graph of \( y = x^2 + \frac{2}{x} \) for \( 0.1 \leq x \leq 4 \)

(c) Use your graph to find estimates for the solutions of \( x^2 + \frac{2}{x} = 14 \)

in the interval \( 0.1 \leq x \leq 4 \)

Give your estimates correct to 1 decimal place.
(d) \( x = 1 \) is one solution of the equation \( x^2 + \frac{2}{x} = mx \)

(i) Find the value of \( m \).

\[ m = \ldots \ldots \ldots \ldots \]

(ii) Draw a suitable straight line on your graph to find an estimate for the second positive solution of the equation \( x^2 + \frac{2}{x} = mx \) for the value of \( m \) found in part (d)(i).
Give your estimate correct to 1 decimal place.

\[ x = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]

(3)

(Total for Question 20 is 8 marks)

Do NOT write in this space.
A garage tests cars for faults.
There are three types of fault – braking, steering and lighting.
A car fails the test if it has one or more of these three types of fault.

Last week, 11 cars had braking faults
9 cars had steering faults
7 cars had lighting faults
no car had both steering faults and lighting faults
2 cars had both braking faults and steering faults
3 cars had both braking faults and lighting faults.

By drawing a Venn Diagram, or otherwise, find the number of cars which failed the test last week.
The diagram shows a regular hexagon. The perimeter of the hexagon is 42 cm.

Calculate the area of the hexagon. Give your answer correct to 3 significant figures.

\[ \text{Area} = \text{cm}^2 \]

(Total for Question 22 is 5 marks)
23 (a) Show that \( \frac{x^2 + 3x}{2x^2 + 5x - 3} \) can be written as \( \frac{x}{kx - 1} \)

State the value of \( k \).

\[ k = \underline{\text{\hspace{1cm}}} \]  

(2)

(b) \( f(x) = \frac{x}{2x - 1} \)

Find the inverse function \( f^{-1} \) in the form \( f^{-1}(x) = \ldots \)

Show your working clearly.

\[ f^{-1}(x) = \underline{\text{\hspace{1cm}}} \]  

(3)

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS