Edexcel IGCSE

Mathematics A

Paper 3H

Higher Tier

Monday 6 June 2011 – Afternoon

Time: 2 hours

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Without sufficient working, correct answers may be awarded no marks.
• Answer the questions in the spaces provided – there may be more space than you need.
• Calculators may be used.
• You must NOT write anything on the formulae page.
   Anything you write on the formulae page will gain NO credit.

Information

• The total mark for this paper is 100.
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Check your answers if you have time at the end.

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IGCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

Pythagoras’ Theorem

$\sqrt{a^2 + b^2} = c$

Volume of cone = $\frac{1}{3} \pi r^2 h$

Volume of sphere = $\frac{4}{3} \pi r^3$

Curved surface area of cone = $\pi rl$

Surface area of sphere = $4 \pi r^2$

adj = hyp $\times \cos \theta$

hyp

opp = hyp $\times \sin \theta$

opp = adj $\times \tan \theta$

adj

In any triangle $ABC$

$\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$\cos \theta = \frac{\text{adj}}{\text{hyp}}$

$\tan \theta = \frac{\text{opp}}{\text{adj}}$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

Volume of prism = area of cross section $\times$ length

Area of a trapezium = $\frac{1}{2} (a + b)h$

Circumference of circle = $2 \pi r$

Area of circle = $\pi r^2$

Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2 \pi rh$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1  (a) Use your calculator to work out the value of

\[
\frac{24.1}{8.4 - 7.8} - 6.2^2
\]

Write down all the figures on your calculator display.

(2)

(b) Give your answer to part (a) correct to 3 significant figures.

(1)

(Total for Question 1 is 3 marks)
Four of the angles of a pentagon are $97^\circ$, $114^\circ$, $127^\circ$ and $84^\circ$.

Work out the size of the fifth angle.

\[\boxed{\phantom{0000}}\] °

(Total for Question 2 is 4 marks)
3  (a) Factorise  \(w^2 - 9w\).

(b) Solve  \(5x - 1 = 2x - 7\)

\[x = \ldots\]

(c) Expand and simplify  \((y - 7)(y + 3)\).

\[\ldots\]

(Total for Question 3 is 7 marks)
Every morning, Samath has one glass of fruit juice with his breakfast. He chooses at random orange juice or pineapple juice or mango juice.
The probability that he chooses orange juice is 0.6
The probability that he chooses pineapple juice is 0.3

(a) Work out the probability that he chooses mango juice.

(b) There are 30 days in April.

Work out an estimate for the number of days in April on which Samath chooses orange juice.

Show that \( \frac{5}{6} - \frac{3}{4} = \frac{1}{12} \)

(Total for Question 4 is 4 marks)

(Total for Question 5 is 2 marks)
6

(a) Describe fully the single transformation which maps triangle P onto triangle Q.

.......................................................................................................................... ...
..........................................................................................................................

(b) Reflect triangle Q in the line \( y = x \).

Label the new triangle R.

(Total for Question 6 is 5 marks)

7

The perimeter of a triangle is 90 cm.

The lengths of the sides of the triangle are in the ratios 3 : 5 : 7

Work out the length of the longest side of the triangle.

.............................. cm

(Total for Question 7 is 3 marks)
8 \( \mathcal{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)
\( A = \{\text{odd numbers}\} \)
\( P = \{\text{prime numbers}\} \)

List the members of the set

(i) \( A \cap P \),

..............................................................................................

(ii) \( A \cup P \),

..............................................................................................

(Total for Question 8 is 2 marks)

9 Ella invested $8000 for 3 years at 5% per annum compound interest.

Calculate the value of her investment at the end of 3 years.

$ ............................................

(Total for Question 9 is 3 marks)
This rule can be used to work out the fare, in dirhams, for a taxi journey in Dubai.

Find a formula for the fare, $C$ dirhams, for a taxi journey of $d$ kilometres.

- Multiply the number of kilometres travelled by 3
- Add 7 to your result
- Then divide by 2
- Taxi fare in dirhams

Find a formula for the fare, $C$ dirhams, for a taxi journey of $d$ kilometres.

(Total for Question 10 is 3 marks)
The table shows information about the weights of 80 parcels.

<table>
<thead>
<tr>
<th>Weight (w kg)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; w ≤ 2</td>
<td>8</td>
</tr>
<tr>
<td>2 &lt; w ≤ 4</td>
<td>14</td>
</tr>
<tr>
<td>4 &lt; w ≤ 6</td>
<td>26</td>
</tr>
<tr>
<td>6 &lt; w ≤ 8</td>
<td>17</td>
</tr>
<tr>
<td>8 &lt; w ≤ 10</td>
<td>10</td>
</tr>
<tr>
<td>10 &lt; w ≤ 12</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Work out an estimate for the total weight of the 80 parcels.

(b) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Weight (w kg)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; w ≤ 2</td>
<td></td>
</tr>
<tr>
<td>0 &lt; w ≤ 4</td>
<td></td>
</tr>
<tr>
<td>0 &lt; w ≤ 6</td>
<td></td>
</tr>
<tr>
<td>0 &lt; w ≤ 8</td>
<td></td>
</tr>
<tr>
<td>0 &lt; w ≤ 10</td>
<td></td>
</tr>
<tr>
<td>0 &lt; w ≤ 12</td>
<td></td>
</tr>
</tbody>
</table>
(c) On the grid, draw a cumulative frequency graph for your table.

(d) Use the graph to find an estimate for the number of parcels which weighed less than 5.2 kg.

(Total for Question 11 is 8 marks)
$AB$ is parallel to $DE$.
$ACE$ and $BCD$ are straight lines.
$AB = 9$ cm.
$AC = 7.2$ cm.
$CD = 5.2$ cm.
$DE = 6$ cm.

(a) Calculate the length of $BC$.

(b) Calculate the length of $CE$.

(Total for Question 12 is 4 marks)
13 Solve \( \frac{2x-1}{4} + \frac{x-1}{5} = 2 \)

\[
x = \ldots
\]

(Total for Question 13 is 4 marks)

14 \( y = 1.8 \) correct to 1 decimal place.

Calculate the lower bound for the value of \( 4y + 1 \)

\[
\ldots
\]

(Total for Question 14 is 2 marks)
Here is a shape made from a rectangle and a semicircle.

The length of the rectangle is 7.1 cm.
The radius of the semicircle is 2.7 cm.

Work out the area of the shape.
Give your answer correct to 3 significant figures.

........................................ cm²

(4)
(b) Here is another shape made from a rectangle and a semicircle.

The length of the rectangle is $L$ cm.
The radius of the semicircle is $r$ cm.

The perimeter, $P$ cm, of the shape is given by the formula

$$P = \pi r + 2L + 2r$$

Make $r$ the subject of the formula $P = \pi r + 2L + 2r$.

$$r = \frac{P - 2L}{\pi + 2}$$

(Total for Question 15 is 7 marks)
$A, B, C$ and $D$ are points on a circle, centre $O$.
Angle $ADB = 57^\circ$.
Angle $BCD = 106^\circ$.

(a) (i) Calculate the size of angle $AOB$.

(ii) Give a reason for your answer.

(b) Calculate the size of angle $BAD$.

(Total for Question 16 is 3 marks)
Here are seven counters.
Each counter has a number on it.

Ali puts the seven counters in a bag.
He takes, at random, a counter from the bag and does not replace the counter.
He then takes, at random, a second counter from the bag.

Calculate the probability that

(i) the number on the second counter is 2 more than the number on the first counter,

(ii) the number on the second counter is 1 more than the number on the first counter.

(Total for Question 17 is 5 marks)
Triangle $ABC$ is right-angled at $B$.
Angle $BAC = 32^\circ$.
$AC = 47$ m.
$D$ is the point on $AB$ such that angle $BDC = 51^\circ$.

Calculate the length of $BD$.
Give your answer correct to 3 significant figures.

\[
\quad
\text{(Total for Question 18 is 5 marks)}
\]
19 \( P \) is directly proportional to the cube of \( Q \).
When \( Q = 15, \ P = 1350 \)
(a) Find a formula for \( P \) in terms of \( Q \).

\[
P = \frac{Q^3 \times a}{10^n}
\]

(b) Calculate the value of \( P \) when \( Q = 20 \)

\[
P = \frac{20^3 \times a}{10^n}
\]

(Total for Question 19 is 4 marks)

20 \( x = a \times 10^n \) where \( n \) is an integer and \( 10 \leq a < 10 \)

Find, in standard form, an expression for \( x^2 \).
Give your expression as simply as possible.

\[
(\text{Total for Question 20 is 3 marks})
\]
$ABCD$ is a rectangle.
$AB = 10$ cm.
$BC = 8$ cm.
$P$, $Q$, $R$ and $S$ are points on the sides of the rectangle.
$BP = CQ = DR = AS = x$ cm.

(a) Show that the area, $A$ cm$^2$, of the quadrilateral $PQRS$ is given by the formula

$$A = 2x^2 - 18x + 80$$

Diagram NOT accurately drawn
(b) For \( A = 2x^2 - 18x + 80 \)

(i) find \( \frac{dA}{dx} \),

(ii) find the value of \( x \) for which \( A \) is a minimum.

\( x = \) ........................................

(iii) Explain how you know that \( A \) is a minimum for this value of \( x \).

.......................................................................................................................... ...

.......................................................................................................................... ...

(5)

(Total for Question 21 is 8 marks)
22 Solve the simultaneous equations

\[ y = 2x - 3 \]
\[ x^2 + y^2 = 2 \]

(Total for Question 22 is 6 marks)
The diagram shows a solid cylinder and a solid sphere.
The cylinder has radius $r$.
The sphere has radius $r$.

Given that $\frac{\text{Total surface area of cylinder}}{\text{Surface area of sphere}} = 2$

find the value of $\frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$

(Total for Question 23 is 5 marks)