Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
**International GCSE MATHEMATICS**  
**FORMULAE SHEET – HIGHER TIER**

**Pythagoras' Theorem**

\[ a^2 + b^2 = c^2 \]

**Volume of cone** = \( \frac{1}{3} \pi r^2 h \)

**Curved surface area of cone** = \( \pi rl \)

**Volume of sphere** = \( \frac{4}{3} \pi r^3 \)

**Surface area of sphere** = \( 4 \pi r^2 \)

**In any triangle** \( ABC \)

- \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \)
- \( \cos \theta = \frac{\text{adj}}{\text{hyp}} \)
- \( \tan \theta = \frac{\text{opp}}{\text{adj}} \)

**Sine rule:**
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

**Cosine rule:**
\[ a^2 = b^2 + c^2 - 2bc \cos A \]

**Area of triangle** = \( \frac{1}{2} ab \sin C \)

**Volume of prism** = area of cross section \( \times \) length

**Circumference of circle** = \( 2 \pi r \)

**Area of circle** = \( \pi r^2 \)

**Volume of cylinder** = \( \pi r^2 h \)

**Curved surface area** of cylinder = \( 2 \pi rh \)

**The Quadratic Equation**

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
A box contains some coloured cards.
Each card is red or blue or yellow or green.
The table shows the probability of taking a red card or a blue card or a yellow card.

<table>
<thead>
<tr>
<th>Card</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>0.3</td>
</tr>
<tr>
<td>Blue</td>
<td>0.35</td>
</tr>
<tr>
<td>Yellow</td>
<td>0.15</td>
</tr>
<tr>
<td>Green</td>
<td></td>
</tr>
</tbody>
</table>

George takes at random a card from the box.

(a) Work out the probability that George takes a green card.

......................................................
(2)

George replaces his card in the box.
Anish takes a card from the box and then replaces the card.
Anish does this 40 times.

(b) Work out an estimate for the number of times Anish takes a yellow card.

......................................................
(2)

(Total for Question 1 is 4 marks)
2 Wendy travelled on the Eurostar train from St Pancras station to the Gare du Nord station. The Eurostar train travelled a distance of 495 km. The journey time was 2 hours 15 minutes.

Work out the average speed of the Eurostar train in kilometres per hour.

\[
\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{495 \text{ km}}{2 \text{ hours } 15 \text{ minutes}} = \frac{495 \text{ km}}{2.25 \text{ hours}}
\]

\[
\text{Average speed} = 219.33 \text{ km/h}
\]

(Total for Question 2 is 3 marks)

3 The table shows information about the time, in minutes, spent on homework by each of 32 pupils in one night.

<table>
<thead>
<tr>
<th>Time (t minutes)</th>
<th>Number of pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; t &lt; 20</td>
<td>7</td>
</tr>
<tr>
<td>20 &lt; t &lt; 40</td>
<td>16</td>
</tr>
<tr>
<td>40 &lt; t &lt; 60</td>
<td>3</td>
</tr>
<tr>
<td>60 &lt; t &lt; 80</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Calculate the percentage of the 32 pupils who spent more than 60 minutes on their homework.

\[
\text{Percentage} = \left( \frac{6 + 3}{32} \right) \times 100 = \left( \frac{9}{32} \right) \times 100 = 28.125\%
\]

(b) Calculate an estimate for the total time spent on homework by the 32 pupils.

\[
\text{Total time} = (7 \times 20) + (16 \times 40) + (3 \times 60) + (6 \times 80) = 140 + 640 + 180 + 480 = 1440 \text{ minutes}
\]

(Total for Question 3 is 5 marks)
4 (a) Expand \( 6(3a - 2b + c) \)

(b) Factorise \( t^2 - 10t \)

(c) Solve \( x = \frac{7 - 2x}{3} \)

Show clear algebraic working.

\[ x = \ldots \]

(Total for Question 4 is 6 marks)

5 Show that \( \frac{4}{9} - \frac{1}{6} = \frac{5}{18} \)

(Total for Question 5 is 2 marks)
(a) Describe fully the single transformation that maps shape P onto shape Q.

(b) On the grid, rotate shape Q 180° about the point (6, 2).
Label the new shape R.

(Total for Question 6 is 5 marks)
7 \[ M = 3x^2 - nx \]

(a) Work out the value of \( M \) when
\[ x = -2 \text{ and } n = 5 \]

\[ M = \]
\[ ...................................................... \]  
(2)

(b) Work out the value of \( n \) when
\[ M = 12 \text{ and } x = 4 \]

\[ n = \]
\[ ...................................................... \]  
(3)

(Total for Question 7 is 5 marks)

Do NOT write in this space.
8 (a) \( A = \{s, u, p, e, r\} \)
\( B = \{c, o, m, p, u, t, e, r\} \)

List the members of the set
(i) \( A \cap B \)
........................................................................................................
(ii) \( A \cup B \)
........................................................................................................ (2)

(b) \( X = \{\text{prime numbers}\} \)
\( Y = \{\text{factors of 12}\} \)

Is it true that \( X \cap Y = \emptyset \)?

Tick (✓) the appropriate box.

Yes  No

Explain your answer.
..................................................................................................................................................................................................................................................

(Total for Question 8 is 3 marks)

9 (a) Simplify, leaving your answers in index form,
(i) \( 6^5 \times 6^2 \times 6 \)
........................................................................................................
(ii) \( (9^7)^2 \)
........................................................................................................ (2)

(b) \( \frac{5^n \times 5^3}{5^6} = 5^4 \)

Find the value of \( n \).

\( n = \) ........................................................................................................ (2)

(Total for Question 9 is 4 marks)
The diagram shows the path of an athlete on a running track.

The path consists of two straight lengths and a semicircle at each end.
Each straight length is 85 metres.
Each semicircle has a radius of 36.6 metres.

Calculate the area enclosed by the path.
Give your answer correct to 3 significant figures.

...................................................... m²

(Total for Question 10 is 4 marks)
11 Calculate the value of $x$.
Give your answer correct to 3 significant figures.

$$x = \ldots$$

(Total for Question 11 is 3 marks)

12 (a) Write as an ordinary number

(i) $4.2 \times 10^6$

(ii) $3.82 \times 10^{-4}$

(b) Here are three numbers written in standard form.
Arrange these numbers in order of size. Start with the smallest number.

$$5.6 \times 10^{-7} \quad 8.6 \times 10^{-9} \quad 5.64 \times 10^{-8}$$

(Total for Question 12 is 4 marks)
The points with coordinates (0, 3) and (8, 5) lie on the straight line $L$.

(a) Work out the gradient of $L$.

(b) Write down an equation of $L$.

(c) Find an equation of the line which is parallel to $L$ and which passes through the point $(-4, -2)$.

(Total for Question 13 is 5 marks)
14 Triangles $ABC$ and $ACD$ are similar.

Angle $BAC = \text{angle } CAD$.
Angle $ABC = \text{angle } ACD$.
$AB = 5\text{ cm}$ and $AC = 8\text{ cm}$.

(a) Calculate the length of $AD$.

\[..........................\text{ cm}\]

(2)

The area of triangle $ABC$ is $12\text{ cm}^2$

(b) Calculate the area of triangle $ACD$.

\[..........................\text{ cm}^2\]

(2)

(Total for Question 14 is 4 marks)
15 The table shows information about the times, in minutes, that some people took to complete a sudoku puzzle.

<table>
<thead>
<tr>
<th>Time (t minutes)</th>
<th>0 &lt; t ≤ 5</th>
<th>5 &lt; t ≤ 20</th>
<th>20 &lt; t ≤ 30</th>
<th>30 &lt; t ≤ 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people</td>
<td>4</td>
<td>18</td>
<td>34</td>
<td>30</td>
</tr>
</tbody>
</table>

Complete the histogram for this information.

(Total for Question 15 is 3 marks)
Glass A contains 122 millilitres of water, correct to the nearest millilitre. Glass B contains 168 millilitres of water, correct to the nearest millilitre.

Calculate the upper bound of the difference, in millilitres, between the volume of water in glass A and the volume of water in glass B.

...................................................... millilitres

(Total for Question 16 is 2 marks)

17 Make $n$ the subject of the formula

\[ t = \sqrt{\frac{n + 3}{n}} \]

......................................................

\[ n = \]

(Total for Question 17 is 4 marks)
Boris and Nigel play games of chess against each other in a match. In each game, Boris wins or Nigel wins or the game is a draw.

When a player wins a game, he wins the match. When a game is a draw, the players play another game against each other. Boris and Nigel play a maximum of 3 games.

The probability that Boris wins a game is \( \frac{1}{3} \)

The probability that a game is a draw is \( \frac{1}{2} \)

(a) Complete the probability tree diagram.

(b) Calculate the probability that Boris wins the match.
19 A particle is moving in a straight line which passes through a fixed point \( O \).
The displacement, \( s \) metres, of the particle from \( O \) at time \( t \) seconds is given by
\[
s = 10 + 9t^2 - t^3
\]
(a) Find an expression for the velocity, \( v \) m/s, of the particle at time \( t \) seconds.

\[
v = \frac{ds}{dt} = 18t - 3t^2
\]

(2)

(b) Find the time at which the acceleration of the particle is zero.

\[
\frac{dv}{dt} = 18 - 6t = 0 \Rightarrow t = 3
\]

(2)

(Total for Question 19 is 4 marks)

20 \( PTR \) and \( QTS \) are chords of a circle.

\( PT = 3 \) cm.
\( ST = 10 \) cm.
\( RT = 15 \) cm.
\( QT = x \) cm.

Diagram NOT accurately drawn

Calculate the value of \( x \).

\[
x = \sqrt{10^2 + 15^2} - 3 = \sqrt{225 + 100} - 3 = \sqrt{325} - 3
\]

(2)

(Total for Question 20 is 2 marks)
21 A bag contains $x$ counters.
    7 of the counters are blue.
    Sam takes at random a counter from the bag and does not replace it.
    Jill then takes a counter from the bag.
    The probability they both take a blue counter is 0.2

(a) Form an equation involving $x$.
    Show that your equation can be expressed as $x^2 - x - 210 = 0$

(b) Solve $x^2 - x - 210 = 0$
    Show clear algebraic working.

22 $(\sqrt{a} + \sqrt{8a})^2 = 54 + b\sqrt{2}$
    $a$ and $b$ are positive integers.
    Find the value of $a$ and the value of $b$.
    Show your working clearly.

    $a =$ ......................................................
    $b =$ ......................................................

(Total for Question 21 is 5 marks)

(Total for Question 22 is 3 marks)
$OABC$ is a parallelogram.
$BCD$ is a straight line.
$BD = 3BC$.
$M$ is the midpoint of $OC$.
\[ \overrightarrow{OA} = x \quad \overrightarrow{AB} = y \]

(a) Find, in terms of $x$ and $y$,

(i) $\overrightarrow{AM}$

(ii) $\overrightarrow{OD}$

(b) Use your answers to (a)(i) and (ii) to write down two different geometric facts about the lines $AM$ and $OD$.

(Total for Question 23 is 4 marks)
The diagram shows a cube $ABCDEFGH$. The sides of the cube are of length 5 cm.

Calculate the size of the angle between the diagonal $AH$ and the base $EFGH$. Give your answer correct to 1 decimal place.

(Total for Question 24 is 4 marks)
25 Solve the simultaneous equations

\[x^2 + y^2 = 26\]
\[y = 3 - 2x\]

Show clear algebraic working.

(Total for Question 25 is 6 marks)