International GCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

Pythagoras’ Theorem

\[ a^2 + b^2 = c^2 \]

Volume of cone = \( \frac{1}{3} \pi r^2 h \)
Curved surface area of cone = \( \pi rl \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)
Surface area of sphere = \( 4\pi r^2 \)

Theorem
\[ a^2 + b^2 = c^2 \]

adj = hyp \times \cos \theta
opp = hyp \times \sin \theta
opp = adj \times \tan \theta

or \[ \sin \theta = \frac{\text{opp}}{\text{hyp}} \]
\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} \]
\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]

In any triangle \( ABC \)

Sine rule: \[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
Cosine rule: \[ a^2 = b^2 + c^2 - 2bc \cos A \]
Area of triangle = \( \frac{1}{2} \ ab \ \sin C \)

Volume of prism = area of cross section \times length

Area of a trapezium = \( \frac{1}{2} (a + b)h \)

Circumference of circle = \( 2\pi r \)
Area of circle = \( \pi r^2 \)

Volume of cylinder = \( \pi r^2 h \)
Curved surface area of cylinder = \( 2\pi rh \)

The Quadratic Equation
The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1  The table shows information about the numbers of fish caught by 40 people in one day.

<table>
<thead>
<tr>
<th>Number of fish</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Work out the mean number of fish caught.

...............................

(3)

(b) Work out what percentage of the 40 people caught less than 2 fish.

...............................

%  

(2)

(Total for Question 1 is 5 marks)
2 Each exterior angle of a regular polygon is $15^\circ$.
   
   (a) How many sides has the regular polygon?

   .............................................

   (2)

   The diagram shows 3 identical regular pentagons.

   (b) Work out the value of $y$.

   $y = \underline{\hspace{2cm}}$

   (3)

   (Total for Question 2 is 5 marks)
3 Use your calculator to work out the value of

\[
\frac{12.5 \times 4.5}{6.8 + \sqrt{67.24}}
\]

(Total for Question 3 is 2 marks)

4 Solve \(7x - 2 = 1 - 3x\)
Show clear algebraic working.

\(x = \ldots\)

(Total for Question 4 is 3 marks)
(a) Describe fully the single transformation which maps shape P onto shape Q.

(b) Reflect the shape Q in the line y = x.
Label the new shape R.
(c) Enlarge shape $S$ with scale factor $\frac{1}{2}$ and centre $(1, 3)$

(Total for Question 5 is 6 marks)
6. The mean height of a group of 6 children is 165 cm. One child, whose height is 155 cm, leaves the group. Find the mean height of the remaining 5 children.

\[ \text{Total for Question 6 is 3 marks} \]

\[ \text{cm} \]

7. Work out the value of \( x \). Give your answer correct to 3 significant figures.

\[ x = \text{.................} \]

\[ \text{(Total for Question 7 is 3 marks)} \]
8  (a) Factorise $g^2 + 4g$

............................................................................

(2)

(b) Factorise $e^2 - 2e - 24$

............................................................................

(2)

(Total for Question 8 is 4 marks)

9  Make $r$ the subject of the formula $A = 4\pi r^2$ where $r$ is positive.

$r =$

(Total for Question 9 is 2 marks)
10 (a) \( A = 2^2 \times 3 \times 5^2 \)

\( B = 2^3 \times 5 \)

(i) Find the Highest Common Factor (HCF) of \( A \) and \( B \).

(ii) Find the Lowest Common Multiple (LCM) of \( A \) and \( B \).

(b) \( \frac{8^2 \times 8^3}{8^4} = 2^n \)

Find the value of \( n \).

\( n = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS (Total for Question 10 is 5 marks)
The diagram shows a right-angled triangle and a rectangle.

(i) Write down an equation for $x$.

(ii) Find the area of the rectangle.
    Show clear algebraic working.

(Total for Question 11 is 7 marks)
The grouped frequency table gives information about the times recorded for 20 runners in a 1500 metre race.

<table>
<thead>
<tr>
<th>Time ($t$ seconds)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$225 &lt; t \leq 230$</td>
<td>1</td>
</tr>
<tr>
<td>$230 &lt; t \leq 235$</td>
<td>3</td>
</tr>
<tr>
<td>$235 &lt; t \leq 240$</td>
<td>7</td>
</tr>
<tr>
<td>$240 &lt; t \leq 245$</td>
<td>6</td>
</tr>
<tr>
<td>$245 &lt; t \leq 250$</td>
<td>2</td>
</tr>
<tr>
<td>$250 &lt; t \leq 255$</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Time ($t$ seconds)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$225 &lt; t \leq 230$</td>
<td></td>
</tr>
<tr>
<td>$225 &lt; t \leq 235$</td>
<td></td>
</tr>
<tr>
<td>$225 &lt; t \leq 240$</td>
<td></td>
</tr>
<tr>
<td>$225 &lt; t \leq 245$</td>
<td></td>
</tr>
<tr>
<td>$225 &lt; t \leq 250$</td>
<td></td>
</tr>
<tr>
<td>$225 &lt; t \leq 255$</td>
<td></td>
</tr>
</tbody>
</table>
(b) On the grid, draw the cumulative frequency graph for your table.

(c) Use your graph to find an estimate for the median of the recorded times.

(seconds)

(Total for Question 12 is 5 marks)
The table shows information about the oil production, in barrels per day, of five countries during one year.

<table>
<thead>
<tr>
<th>Country</th>
<th>Oil production (barrels per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>India</td>
<td>$8.97 \times 10^5$</td>
</tr>
<tr>
<td>Brazil</td>
<td>$2.63 \times 10^6$</td>
</tr>
<tr>
<td>United States</td>
<td>$8.4 \times 10^6$</td>
</tr>
<tr>
<td>Russia</td>
<td>$1.09 \times 10^7$</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>$9.9 \times 10^6$</td>
</tr>
</tbody>
</table>

(a) Which country had the highest oil production?

........................................................................................................

(1)

(b) Calculate the difference between the oil production of Brazil and the oil production of India. Give your answer in standard form.

........................................................................................................ barrels per day

(2)

During the same year, the oil production of California was $6.3 \times 10^5$ barrels per day.

(c) Work out the oil production of California as a proportion of the oil production of the United States.

........................................................................................................

(2)

(Total for Question 13 is 5 marks)
14 Solve the simultaneous equations

\[ 8x - 4y = 7 \]
\[ 12x - 8y = 6 \]

Show clear algebraic working.

\[ x = \ldots \]
\[ y = \ldots \]

(Total for Question 14 is 3 marks)

15 Use algebra to show that the recurring decimal \[ 0.\overset{\ldots}{4}17 \overset{\ldots}{3}33 = \frac{139}{333} \]

(Total for Question 15 is 2 marks)
16 \(ABCD\) is a kite.

\(\overline{AB} = 3\) cm \\
\(\overline{BC} = 8\) cm \\
Angle \(\angle ABC = 110^\circ\)

Calculate the area of the kite \(ABCD\). 
Give your answer correct to 3 significant figures.

\[
\text{Area} = \frac{1}{2} \times \overline{AB} \times \overline{BC} \times \sin \angle ABC
\]

\[
= \frac{1}{2} \times 3 \times 8 \times \sin 110^\circ
\]

\[
= \frac{1}{2} \times 3 \times 8 \times 0.9397
\]

\[
= 11.9168 \approx 11.92 \text{ cm}^2
\]

(Total for Question 16 is 3 marks)
Two bags contain discs.

Bag A contains 12 discs.  
5 of the discs are red, 6 are blue and 1 is white.

Bag B contains 25 discs.  
n of the discs are red and the rest are blue.

James takes at random a disc from Bag A.  
Lucy takes at random a disc from Bag B.

Given that the probability that James and Lucy both take a red disc is \( \frac{2}{15} \)

(i) find the value of \( n \), the number of red discs in Bag B.

\( n = \) ............................................

(ii) Hence calculate the probability that James and Lucy take discs of different colours.

(Total for Question 17 is 5 marks)
PQR is a triangle.
The midpoint of PQ is W.
X is the point on QR such that QX : XR = 2 : 1
PRY is a straight line.
\[ \overrightarrow{PW} = \mathbf{a} \quad \overrightarrow{PR} = \mathbf{b} \]

(a) Find, in terms of \( \mathbf{a} \) and \( \mathbf{b} \),

(i) \( \overrightarrow{QR} \)

(ii) \( \overrightarrow{QX} \)

(iii) \( \overrightarrow{WX} \)

R is the midpoint of the straight line PRY.

(b) Use a vector method to show that WXY is a straight line.
19 The diagram shows a circular pond, of radius \( r \) metres, surrounded by a circular path. The circular path has a constant width of 1.5 metres.

The area of the path is \( \frac{1}{10} \) the area of the pond.

(a) Show that \( 2r^2 - 60r - 45 = 0 \)

(b) Calculate the area of the pond.
Show your working clearly.
Give your answer correct to 3 significant figures.
20 The diagram shows parts of the graphs of $y = f(x)$ and $y = g(x)$.

(a) Find $g(0)$ 
...............................................................
(1)

(b) Find $g(f(-1))$ 
...............................................................
(2)

(c) Calculate an estimate for the gradient of the curve $y = f(x)$ at the point on the curve where $x = 3$ 
...............................................................
(3)

(Total for Question 20 is 6 marks)
21 Correct to 2 significant figures, \( a = 58, \ b = 28 \) and \( c = 18 \)

Calculate the upper bound for the value of \( \frac{a}{b-c} \)

Show your working clearly.

22 Simplify fully \( \frac{6x^2 + x - 15}{12x^2 - 27} \)

Show clear algebraic working.
A, B and C are points on horizontal ground. B is due North of A and AB is 14 m. C is due East of A and AC is 25 m.

A vertical flagpole, TX, has its base at the point X on BC such that the angle AXC is a right angle.

The height of the flagpole, TX, is 10 m.

Calculate the size of the angle of elevation of T from A. Give your answer correct to 1 decimal place.

(Total for Question 23 is 6 marks)
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