Mathematics A
Paper 3H
Higher Tier

Tuesday 6 January 2015 – Afternoon
Time: 2 hours

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- Calculators may be used.
- You must NOT write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
International GCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

Pythagoras’ Theorem

\[ a^2 + b^2 = c^2 \]

Volume of cone = \( \frac{1}{3} \pi r^2 h \)

Curved surface area of cone = \( \pi rl \)

Volume of prism = area of cross section \( \times \) length

Volume of sphere = \( \frac{4}{3} \pi r^3 \)

Surface area of sphere = \( 4 \pi r^2 \)

Theorem

\[ a^2 + b^2 = c^2 \]

adj = hyp \( \times \) cos \( \theta \)

opp = hyp \( \times \) sin \( \theta \)

opp = adj \( \times \) tan \( \theta \)

or \[ \sin \theta = \frac{\text{opp}}{\text{hyp}} \]

\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} \]

\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]

Area of triangle

\[ \frac{1}{2} bc \sin C \]

Area of a trapezium = \( \frac{1}{2}(a + b)h \)

In any triangle \( ABC \)

Sine rule:

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Cosine rule:

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Area of triangle = \( \frac{1}{2} ab \sin C \)

Circumference of circle = \( 2\pi r \)

Area of circle = \( \pi r^2 \)

Volume of cylinder = \( \pi r^2 h \)

Curved surface area of cylinder = \( 2\pi rh \)

The Quadratic Equation

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

<table>
<thead>
<tr>
<th>1 euro = 120 yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>£1 = 1.2 euros</td>
</tr>
</tbody>
</table>

Change £50 to yen.

.............................................. yen

(Total for Question 1 is 2 marks)

2 (a) Work out the value of \( \frac{451.4}{14.1 + 10.3} \)

..............................................

(2)

(b) Work out the value of \( \sqrt{7.8^2 - 7.2^2} \)

..............................................

(2)

(Total for Question 2 is 4 marks)

Do NOT write in this space.
3 (a) Factorise $14x - 35$

(b) Expand and simplify $3(2c - 5) - 2(c - 4)$

(c) Simplify $(4c^3)^2$

(d) Expand and simplify $(a + 5)(2a - 1)$

(Total for Question 3 is 7 marks)
The diagram shows a shape with one line of symmetry.

Work out the area of the shape.

\[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]

\[ \text{Area} = \frac{1}{2} \times 15 \text{ cm} \times 10 \text{ cm} \]

\[ \text{Area} = 75 \text{ cm}^2 \]

(Total for Question 4 is 4 marks)
A jar contains coloured beads.
Ajit takes at random a bead from the jar.
The probability that the bead is yellow is 0.08
The probability that the bead is pink is 0.1
The probability that the bead is blue is 0.25

(a) (i) Find the probability that the bead is yellow or blue.

(ii) Find the probability that the bead is neither yellow nor pink.

Ajit replaces the first bead in the jar.
He then takes at random a second bead from the jar.

(b) Find the probability that the first bead is yellow and the second bead is blue.
A second jar contains 100 coloured beads. 20 of these beads are brown. 

Ajit takes at random a bead from the jar. He records the colour of the bead and then returns the bead to the jar. He does this 60 times. 

(c) Work out an estimate for the number of times Ajit records a brown bead. 

\[ y = \] 

6 Solve \(4y - 1 = 3(6y + 7)\) 
Show clear algebraic working.

\[ y = \] 

(Total for Question 6 is 3 marks)
Eloy’s height was 125 cm when his age was 7 years.
His height was 153 cm when his age was 12 years.

(a) Work out the percentage increase in Eloy’s height between the ages of 7 and 12 years.

.............................................

............................................. cm

Eloy’s height at the age of 12 years was 85% of his height at the age of 20 years.

(b) Work out Eloy’s height when his age was 20 years.

............................................. cm

(Total for Question 7 is 6 marks)
ABC is a triangle.
The point D lies on AC.
Angle BDC = 90°
BD = 10 cm, AB = 15 cm and DC = 12.5 cm.

(a) Calculate the length of AD.
Give your answer correct to 3 significant figures.

.............................................. cm

(3)

(b) Calculate the size of angle BCD.
Give your answer correct to 1 decimal place.

..............................................°

(3)

(Total for Question 8 is 6 marks)
9  (a) Find the sum of the interior angles of a polygon with 7 sides.

(b) Work out the value of $x$.
   Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn
10 (a) Find the gradient of the line with equation $3y - 2x = 6$

(b) Find an equation of the line with gradient $-3$ that passes through the point $(2, 5)$.

(Total for Question 10 is 4 marks)
(a) Find the highest common factor (HCF) of 3780 and 3240
Give your answer as a product of prime factors.

\[3780 = 2^2 \times 3^3 \times 5 \times 7\]
\[3240 = 2^3 \times 3^4 \times 5\]

(b) Find the lowest common multiple (LCM) of 3780 and 3240
Give your answer as a product of prime factors.

(Total for Question 11 is 4 marks)
12 Solve the simultaneous equations

\[5y - 4x = 8\]
\[y + x = 7\]

Show clear algebraic working.

\[x = \ldots \]
\[y = \ldots \]

(Total for Question 12 is 3 marks)
The cumulative frequency graph gives information about the intelligence quotients (IQ) of a random sample of 100 adults.

(a) Use the cumulative frequency graph to find an estimate for the number of adults in the sample who have an IQ between 85 and 115

(b) Find an estimate for the upper quartile of the IQ of adults in the sample.

(Total for Question 13 is 4 marks)
$P, Q, R$ and $S$ are points on a circle, centre $O$.
$QS$ is a diameter of the circle.
$QS$ and $PR$ intersect at the point $T$.
$OS = 5\text{ cm}$, $QT = 3\text{ cm}$ and $TR = 6\text{ cm}$.

Work out the length of $PT$. 

\[
\text{.................. cm} 
\]

(Total for Question 14 is 3 marks)
Zane buys mineral water in large bottles and in small bottles. The large bottles are mathematically similar to the small bottles. Large bottles have a height of 32 cm and a volume of 2000 cm$^3$. Small bottles have a volume of 500 cm$^3$.

Work out the height of a small bottle. Give your answer correct to 3 significant figures.

\[
\text{Height of small bottle} = \frac{2000}{500} \times 32 \approx \boxed{128.0} \text{ cm}
\]

(Total for Question 15 is 3 marks)
Gemma has 9 counters.
Each counter has a number on it.

Gemma puts the 9 counters into a bag.
She takes at random a counter from the bag and does not replace the counter.
She then takes at random a second counter from the bag.

(a) Work out the probability that the number on each counter is an even number.

(b) Work out the probability that the number on the first counter added to the number on
the second counter gives an odd number.

(Total for Question 16 is 5 marks)
17. \( P \) is directly proportional to \( q^3 \)
\( P = 270 \) when \( q = 7.5 \)

(a) Find a formula for \( P \) in terms of \( q \)

\[
P = \frac{k}{q^3}
\]

when \( P = 270 \) and \( q = 7.5 \)

\[
k = P \cdot q^3 = 270 \times 7.5^3 = 379687.5
\]

\[
P = \frac{379687.5}{q^3}
\]

(b) Work out the positive value of \( q \) when \( P = q \)

\[
q = \left(\frac{k}{P}\right)^{\frac{1}{3}} = \left(\frac{379687.5}{q}\right)^{\frac{1}{3}}
\]

\[
q = \frac{379687.5}{q^3}
\]

(2)

(Total for Question 17 is 5 marks)
18 \( y = x^3 - 4x^2 + 4x + 3 \)

(a) Find \( \frac{dy}{dx} \)

The diagram shows a sketch of the curve with equation \( y = x^3 - 4x^2 + 4x + 3 \). The point \( P \) is a turning point on the curve.

(b) Work out the coordinates of \( P \).
Show clear algebraic working.

(c) Write down the range of values of \( x \) for which the curve has a negative gradient.

(Total for Question 18 is 8 marks)
The Venn diagram shows all of the elements in sets $A$, $B$ and $E$.

(a) Write down the elements in $A'$

..............................................

(b) Find $n(A \cap B)'$

..............................................

(c) Find the elements in $(A \cap B) \cup (A \cup B)'$

..............................................

$A \cap C = \emptyset$
$B \cup C = \{5, 6, 7, 8, 9\}$
$n(C) = 3$

(d) Write down the elements in $C$.

..............................................

(Total for Question 19 is 4 marks)
20 \( f: x \mapsto 2x^2 + 1 \quad g: x \mapsto \frac{2x}{x - 1} \quad \text{where } x \neq 1 \)

(a) Express the composite function \(gf\) in the form \(gf: x \mapsto \ldots\)
Give your answer as simply as possible.

\(gf: x \mapsto \ldots\)  

(2)

(b) Express the inverse function \(g^{-1}\) in the form \(g^{-1}: x \mapsto \ldots\)

\(g^{-1}: x \mapsto \ldots\)  

(3)

(Total for Question 20 is 5 marks)
A solid cone has a height of 15 cm. 
The volume of the cone is $320\pi \text{ cm}^3$

Work out the curved surface area of the cone. 
Give your answer correct to 3 significant figures.

\[ \text{cm}^2 \]
The diagram shows a triangle $ABC$.
$AB = (2x + 1)$ cm, $AC = (2x - 1)$ cm and $BC = 2\sqrt{7}$ cm.
Angle $BAC = 60^\circ$

Work out the value of $x$.
Show clear algebraic working.

$x =$ .............................................

(Total for Question 22 is 3 marks)
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