Mathematics A
Paper 4H

Higher Tier

Monday 16 January 2012 – Morning
Time: 2 hours

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions
• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Without sufficient working, correct answers may be awarded no marks.
• Answer the questions in the spaces provided
  – there may be more space than you need.
• Calculators may be used.
• You must NOT write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information
• The total mark for this paper is 100.
• The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice
• Read each question carefully before you start to answer it.
• Check your answers if you have time at the end.

Turn over
# International GCSE MATHEMATICS
## FORMULAE SHEET – HIGHER TIER

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c^2 = a^2 + b^2 )</td>
<td>Pythagoras’ Theorem</td>
</tr>
<tr>
<td>( V_{	ext{cone}} = \frac{1}{3} \pi r^2 h )</td>
<td>Volume of cone</td>
</tr>
<tr>
<td>( A_{	ext{cone}} = \pi rl )</td>
<td>Curved surface area of cone</td>
</tr>
<tr>
<td>( V_{	ext{sphere}} = \frac{4}{3} \pi r^3 )</td>
<td>Volume of sphere</td>
</tr>
<tr>
<td>( A_{	ext{sphere}} = 4 \pi r^2 )</td>
<td>Surface area of sphere</td>
</tr>
</tbody>
</table>

In any triangle \( ABC \):

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \theta )</td>
<td>( \frac{\text{opp}}{\text{hyp}} )</td>
</tr>
<tr>
<td>( \cos \theta )</td>
<td>( \frac{\text{adj}}{\text{hyp}} )</td>
</tr>
<tr>
<td>( \tan \theta )</td>
<td>( \frac{\text{opp}}{\text{adj}} )</td>
</tr>
<tr>
<td>( \text{Sine rule:} )</td>
<td>( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} )</td>
</tr>
<tr>
<td>( \text{Cosine rule:} )</td>
<td>( a^2 = b^2 + c^2 - 2bc \cos A )</td>
</tr>
<tr>
<td>Area of triangle</td>
<td>( \frac{1}{2} ab \sin C )</td>
</tr>
</tbody>
</table>

Volume of prism = area of cross section \( \times \) length

Area of trapezium = \( \frac{1}{2} (a + b)h \)

Circumference of circle = \( 2\pi r \)

Area of circle = \( \pi r^2 \)

Volume of cylinder = \( \pi r^2 h \)

Curved surface area of cylinder = \( 2\pi rh \)

The Quadratic Equation

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
Answer ALL TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Work out the value of \( \frac{6.7 - 2.5}{2.8 \times 0.4} \)

Give your answer as a decimal.

..........................................................................

(Total for Question 1 is 2 marks)

2 An aeroplane flew from Qatar to Bahrain.

The distance flown was 135 km.

The average speed was 180 km/h.

Work out the time taken.

Give your answer in minutes.

.............................................. minutes

(Total for Question 2 is 3 marks)

Do NOT write in this space.
3 Solve $7x - 5 = 3x + 2$
   Show your working clearly.

   \[ x = \ldots \ldots \ldots \ldots \]

   (Total for Question 3 is 3 marks)

4 Three positive whole numbers have a median of 7 and a mean of 5
   Find the range of these three numbers.

   \[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]

   (Total for Question 4 is 3 marks)

Do NOT write in this space.
5 On the grid, draw the graph of \( y = 4x - 1 \) from \( x = -2 \) to \( x = 4 \)
(a) There are 32 students in a class.
All the students are either left-handed or right-handed.
The ratio of the number of left-handed students to the number of right-handed students is $1 : 7$

Work out the number of right-handed students.

(b) Sajid makes a scale model of a lorry.
He uses a scale of $1 : 32$
The length of Sajid’s model lorry is $45$ cm.
Chitra makes a scale model of the same lorry.
She uses a scale of $1 : 72$

Work out the length of Chitra’s model lorry.

Do NOT write in this space.
7 Express 200 as a product of powers of its prime factors.

8 \[
\frac{y^3 \times y^n}{y} = y^6
\]

Find the value of \(n\).

\[n = \ldots\]

(Total for Question 8 is 2 marks)

Do NOT write in this space.
"ABCD is a rhombus. The diagonals AC and BD cross at the point E. 
AE = CE = 6 cm.
BE = DE = 4 cm.
Angle AEB = 90°

(a) Work out the area of the rhombus.

\[ \text{Area} = \frac{1}{2} \times 6 \times 4 = \frac{24}{2} = 12 \text{ cm}^2 \]

(b) Work out the length of AB.
Give your answer correct to 3 significant figures.

\[ AB = \sqrt{AE^2 + BE^2} = \sqrt{6^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52} \approx 7.21 \text{ cm} \]

(Total for Question 9 is 6 marks)
10 (i) Solve the inequalities \(-6 < 4x \leq 8\)

(ii) \(n\) is an integer.
Write down all the values of \(n\) which satisfy \(-6 < 4n \leq 8\)

(Total for Question 10 is 4 marks)

11 (a) Find the Highest Common Factor (HCF) of 75 and 90

(b) Find the Lowest Common Multiple (LCM) of 75 and 90

(Total for Question 11 is 4 marks)
(a) Describe fully the single transformation which maps triangle P onto triangle Q.

(b) On the grid, translate triangle Q by the vector \( \begin{pmatrix} 4 \\ -2 \end{pmatrix} \)

Label the new triangle R.

(c) Describe fully the single transformation which maps triangle P onto triangle R.

(Total for Question 12 is 6 marks)
13 (a) Find the gradient of the line with equation \(3x + 4y = 10\)

(b) Find the coordinates of the point of intersection of the line with equation \(3x + 4y = 10\)
and the line with equation \(5x - 6y = 23\)
Show your working clearly.

(Total for Question 13 is 8 marks)
The grouped frequency table gives information about the ages of 200 elephants.

<table>
<thead>
<tr>
<th>Age (t years)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; t \leq 10</td>
<td>55</td>
</tr>
<tr>
<td>10 &lt; t \leq 20</td>
<td>60</td>
</tr>
<tr>
<td>20 &lt; t \leq 30</td>
<td>40</td>
</tr>
<tr>
<td>30 &lt; t \leq 40</td>
<td>22</td>
</tr>
<tr>
<td>40 &lt; t \leq 50</td>
<td>13</td>
</tr>
<tr>
<td>50 &lt; t \leq 60</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Age (t years)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; t \leq 10</td>
<td></td>
</tr>
<tr>
<td>0 &lt; t \leq 20</td>
<td></td>
</tr>
<tr>
<td>0 &lt; t \leq 30</td>
<td></td>
</tr>
<tr>
<td>0 &lt; t \leq 40</td>
<td></td>
</tr>
<tr>
<td>0 &lt; t \leq 50</td>
<td></td>
</tr>
<tr>
<td>0 &lt; t \leq 60</td>
<td></td>
</tr>
</tbody>
</table>
(b) On the grid, draw a cumulative frequency graph for your table.

(c) Use the graph to find an estimate for the number of elephants with ages of more than 26 years.

(Total for Question 14 is 5 marks)
15 Solve the inequality \( x^2 < 16 \)

(Total for Question 15 is 2 marks)

16 Here are 8 dominoes.

The 8 dominoes are put in a bag.

Riaz takes at random a domino from the bag.

(a) Find the probability that he takes a domino with a total of 8 spots or a domino with a total of 9 spots.

(2)
Helima takes at random 2 dominoes from the bag of 8 dominoes without replacement.

(b) Work out the probability that

(i) the total number of spots on the two dominoes is 18

(ii) the total number of spots on the two dominoes is 17

(Total for Question 16 is 7 marks)
17 \[ f(x) = \sqrt{x - 6} \]

(a) Find \( f(10) \) ...........................................

...........................................

...........................................

(b) State which values of \( x \) must be excluded from a domain of \( f \)

The diagram shows part of the graph of \( y = g(x) \)

(c) Find \( g(2) \) ...........................................

...........................................

...........................................
(d) Find \( fg(0) \)

(e) One of the solutions of \( g(x) = k \), where \( k \) is a number, is \( x = 1 \)

Find the other solutions.
Give your answers correct to 1 decimal place.

(f) Find an estimate for the gradient of the curve at the point where \( x = 3.5 \)
Show your working clearly.

(Total for Question 17 is 12 marks)
Calculate the value of $x$.
Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

$x = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$  

(Total for Question 18 is 3 marks)
19 \( A \) and \( B \) are two sets.

\[
\begin{align*}
\text{n}(E) &= 37 \\
\text{n}(A) &= 22 \\
\text{n}(A \cap B) &= 12 \\
\text{n}(A \cup B) &= 30
\end{align*}
\]

(a) Complete the Venn Diagram to show the numbers of elements.

(b) Find

(i) \( \text{n}(A \cap B') \)

(ii) \( \text{n}(A' \cup B') \)

(Total for Question 19 is 4 marks)
A solid cone has a slant height of 9 cm. The **curved** surface area of the cone is 100 cm$^2$.

Calculate the volume of the cone.
Give your answer correct to 3 significant figures.

\[ \text{Volume} = \frac{1}{3} \pi r^2 h \]

\[ r = \text{radius} \\ h = \text{height} \]

\[ \text{Volume} = \frac{1}{3} \pi (\text{radius})^2 (\text{height}) \]

\[ \text{Volume} = \frac{1}{3} \pi (9)^2 (9) \]

\[ \text{Volume} = \frac{1}{3} \pi (81) (9) \]

\[ \text{Volume} = \frac{1}{3} \pi (729) \]

\[ \text{Volume} = 243 \pi \]

\[ \text{Volume} \approx 763.9 \text{ cm}^3 \] (correct to 3 significant figures)

\[ \text{Total for Question 20 is 5 marks} \]
21 (a) Simplify \((16y^8)^{\frac{3}{4}}\)

(b) Given that \(2^p \times 8^q = 2^n\)

express \(n\) in terms of \(p\) and \(q\).

\(n = \ldots\)
OPQR is a rectangle.

D is the point on OP such that $OD = \frac{1}{3} OP$.

E is the point on OQ such that $OE = \frac{2}{3} OQ$.

PQF is the straight line such that $QF = \frac{1}{3} PQ$.

$\overrightarrow{OD} = a$  $\overrightarrow{OR} = 3b$

(a) Find, in terms of $a$ and $b$,

(i) $\overrightarrow{OQ}$

(ii) $\overrightarrow{OE}$

(iii) $\overrightarrow{DE}$

(3)
(b) Use a vector method to prove that DEF is a straight line.
BLANK PAGE

Do NOT write on this page.