London Examinations IGCSE
Mathematics
Paper 4H
Higher Tier
Wednesday 8 November 2006 – Morning
Time: 2 hours

Materials required for examination
Ruler graduated in centimetres and
millimetres, protractor, compasses,
pen, HB pencil, eraser, calculator.
Tracing paper may be used.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initial(s) and
signature.
The paper reference is shown at the top of this page. Check that you have the correct question paper.
Answer ALL the questions in the spaces provided in this question paper.
Show all the steps in any calculations.

Information for Candidates
There are 20 pages in this question paper. All blank pages are indicated.
The total mark for this paper is 100. The marks for parts of questions are shown in round brackets:
e.g. (3).
You may use a calculator.

Advice to Candidates
Write your answers neatly and in good English.

Turn over
IGCSE MATHEMATICS 4400
FORMULA SHEET – HIGHER TIER

Pythagoras’ Theorem

\[ a^2 + b^2 = c^2 \]

Volume of cone = \( \frac{1}{3} \pi r^2 h \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)

Curved surface area of cone = \( \pi rl \)

Surface area of sphere = \( 4 \pi r^2 \)

adj = hyp \times \cos \theta

opp = hyp \times \sin \theta

opp = adj \times \tan \theta

or \[ \sin \theta = \frac{\text{opp}}{\text{hyp}} \]

\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} \]

\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]

In any triangle \( ABC \)

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Sine rule

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Cosine rule

Area of triangle = \( \frac{1}{2} \; ab \sin C \)

Volume of prism = area of cross section \( \times \) length

Area of a trapezium = \( \frac{1}{2} (a + b)h \)

Circumference of circle = \( 2 \pi r \)

Area of circle = \( \pi r^2 \)

Volume of cylinder = \( \pi r^2 h \)

The Quadratic Equation

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY-FIVE questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1. Work out the value of \( \frac{6.46}{1.8+1.6} \)

\[ \\text{\underline{...........................................}} \]

\( \text{(Total 2 marks)} \)

2. (a) Expand \( 3(2t + 5) \)

\[ \text{\underline{..................................................}} \]

\( (1) \)

(b) Expand \( y(y^2 - 3y) \)

\[ \text{\underline{..................................................}} \]

\( (2) \)

(c) Expand and simplify \( (x + 3)(x + 7) \)

\[ \text{\underline{..................................................}} \]

\( (2) \)

(d) Simplify \( p^4q^2 \times p^3q^6 \)

\[ \text{\underline{..................................................}} \]

\( (2) \)

\( \text{Q2} \)

\( \text{(Total 7 marks)} \)
3. The total of Kim’s age and Pablo’s age is 45 years.
The ratio of Kim’s age to Pablo’s age is 1:4

Work out Kim’s age.

............... years

4. Here is a pattern of shapes made from centimetre squares.

<table>
<thead>
<tr>
<th>Shape number 1</th>
<th>Shape number 2</th>
<th>Shape number 3</th>
</tr>
</thead>
</table>

This rule can be used to find the perimeter of a shape in this pattern.

Add 1 to the Shape number and then multiply your answer by 2

$P$ cm is the perimeter of Shape number $n$.

(a) Write down a formula for $P$ in terms of $n$.

..................................................

(b) Make $n$ the subject of the formula in part (a).

$n = ...........................................

(Total 6 marks)
5. Bridget flew from the UK to Dubai. Her flight from the UK to Dubai covered a distance of 5456 km. The flight time was 7 hours 45 minutes.

Work out the average speed of the flight.

\[ \frac{\text{Distance}}{\text{Time}} = \text{Average Speed} \]

\[ \frac{5456 \text{ km}}{7.75 \text{ h}} = \text{km/h} \]

6. \( E = \{ \text{even numbers less than 19} \} \)
   \( M = \{ \text{multiples of 3} \} \)
   \( F = \{ \text{factors of 12} \} \)

(a) (i) Explain why it is **not** true that \( 9 \in M \).

................................................................................................................................

(ii) List the members of \( M \).

..........................................................................................................................

(2)

(b) List the members of \( M \cap F \).

..........................................................................................................................

(2)
7. A solid cylinder has a diameter of 9.4 cm and a height of 8.3 cm.

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

\[ \text{Volume} = \pi r^2 h \]
\[ r = \frac{9.4}{2} = 4.7 \text{ cm} \]
\[ h = 8.3 \text{ cm} \]
\[ \text{Volume} = \pi (4.7)^2 (8.3) \]
\[ \text{Volume} \approx 437.7 \text{ cm}^3 \]

8. \[ y = 4x - 1 \]

Work out the value of \( x \) when \( y = -7 \)

\[ -7 = 4x - 1 \]
\[ 4x = -6 \]
\[ x = -1.5 \]

Q7 (Total 3 marks)
Q8 (Total 2 marks)
9. There are 48 beads in a bag.
Some of the beads are red and the rest of the beads are blue.
Shan is going to take a bead at random from the bag.
The probability that she will take a red bead is \( \frac{3}{8} \)

(a) Work out the number of red beads in the bag.

..........................

(b) Shan adds some red beads to the 48 beads in the bag.
The probability that she will take a red bead is now \( \frac{1}{2} \)

(b) Work out the number of red beads she adds.

..........................

Q9

(Total 4 marks)

10. Express 225 as the product of powers of its prime factors.

..........................

Q10

(Total 3 marks)
(a) Describe fully the single transformation which maps triangle $A$ onto triangle $B$.

......................................................................................................................................
......................................................................................................................................
......................................................................................................................................

(3)

(b) On the grid, translate triangle $A$ by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Label the new triangle $C$.

(Total 5 marks)
12. Solve the simultaneous equations

\[ 6x + 5y = 5 \]
\[ 3x - 10y = 15 \]

\[ x = \ldots \]
\[ y = \ldots \]

13. (a) Write the number 78 000 000 in standard form.

\[ \ldots \quad (1) \]

(b) Write \( 4 \times 10^{-3} \) as an ordinary number.

\[ \ldots \quad (1) \]

(c) Work out the value of \( \frac{3 \times 10^{-3}}{8 \times 10^0} \)

Give your answer in standard form.

\[ \ldots \quad (1) \]

(Total 3 marks)
Triangle $LMN$ is right-angled at $N$.
$MN = 5.4$ cm and $LN = 9.3$ cm.

(a) Work out the size of angle $LMN$.
Give your answer correct to 1 decimal place.

.........................°

(b) (i) Write down the upper bound of the length of $MN$.

....................... cm

(ii) Write down the lower bound of the length of $MN$.

....................... cm
The length, 5.4 cm, of $MN$ and the length, 9.3 cm, of $LN$, are each correct to 2 significant figures.
The line $MN$ is horizontal and the line $LN$ is vertical.

(c) Work out the upper bound for the gradient of the line $LM$.

\[
\text{................. (2) Q14}
\]

(Total 7 marks)

15.

The sides of an equilateral triangle $ABC$ and two regular polygons meet at the point $A$.
$AB$ and $AD$ are adjacent sides of a regular 10-sided polygon.
$AC$ and $AD$ are adjacent sides of a regular $n$-sided polygon.

Work out the value of $n$.

\[
n = \text{................. Q15}
\]

(Total 5 marks)
16. The grouped frequency table gives information about the time spent on the Internet last week by each of 80 students.

<table>
<thead>
<tr>
<th>Time (t hours)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; t \leq 5$</td>
<td>28</td>
</tr>
<tr>
<td>$5 &lt; t \leq 10$</td>
<td>22</td>
</tr>
<tr>
<td>$10 &lt; t \leq 15$</td>
<td>14</td>
</tr>
<tr>
<td>$15 &lt; t \leq 20$</td>
<td>10</td>
</tr>
<tr>
<td>$20 &lt; t \leq 25$</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Time (t hours)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; t \leq 5$</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; t \leq 10$</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; t \leq 15$</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; t \leq 20$</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; t \leq 25$</td>
<td></td>
</tr>
</tbody>
</table>

(b) On the grid, draw the cumulative frequency graph for your table.
(c) Use your graph to find an estimate for the number of students who spent more than 17 hours on the Internet last week.
Show your method clearly.

.................... (2) Q16
(Total 5 marks)

17.

Diagram NOT accurately drawn

The diagram shows a sector of a circle, centre C.
The radius of the circle is 8.2 cm.
The angle at the centre of the circle is 67°.

Calculate the area of the sector.
Give your answer correct to 3 significant figures.

.................. cm² Q17
(Total 3 marks)
18. (a) Complete the table of values for \( y = x^2 - \frac{3}{x} \)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>( y )</td>
<td>-5.75</td>
<td>-2</td>
<td></td>
<td></td>
<td></td>
<td>24.4</td>
</tr>
</tbody>
</table>

(b) On the grid, draw the graph of \( y = x^2 - \frac{3}{x} \) for \( 0.5 \leq x \leq 5 \)
(c) Use your graph to find an estimate for a solution of the equation

\[ x^2 - \frac{3}{x} = 0 \]

\[ x = \ldots \ldots \ldots \ldots \]  

(1)

(d) Draw a suitable straight line on your graph to find an estimate for a solution of the equation

\[ x^2 - 2x - \frac{3}{x} = 0 \]

\[ x = \ldots \ldots \ldots \ldots \]  

(2)

19. Convert the recurring decimal 0.2\(\dot{3}\) to a fraction.
20. $\text{A}, \text{B}, \text{C}$ and $\text{D}$ are points on the circumference of a circle. $\text{AB}$ is a diameter of the circle. Angle $\text{ADC} = 119^\circ$.

(a) (i) Work out the size of angle $\text{ABC}$. 

(ii) Give a reason for your answer.

(b) Work out the size of angle $\text{BAC}$. 

Diagram NOT accurately drawn
21. The unfinished table and histogram show information about the weights, in kg, of some babies.

<table>
<thead>
<tr>
<th>Weight ($w$ kg)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; w \leq 2$</td>
<td></td>
</tr>
<tr>
<td>$2 &lt; w \leq 3.5$</td>
<td>150</td>
</tr>
<tr>
<td>$3.5 &lt; w \leq 4.5$</td>
<td>136</td>
</tr>
<tr>
<td>$4.5 &lt; w \leq 6$</td>
<td></td>
</tr>
</tbody>
</table>

(a) Use the histogram to complete the table. (2)

(b) Use the table to complete the histogram. (1)

(Total 3 marks)
22. Younis spins a biased coin twice.
The probability that it will come down heads both times is 0.36
Calculate the probability that it will come down tails both times.

..........................

Q22
(Total 3 marks)

23. Simplify fully \( \frac{2x^2 - 5x - 12}{4x^2 - 9} \)

..........................

Q23
(Total 3 marks)
Calculate the area of the triangle.
Give your answer correct to 3 significant figures.

\[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]

\[ \text{Area} = \frac{1}{2} \times 8.6 \text{ cm} \times 57^\circ \]

\[ \text{Area} = \frac{1}{2} \times 8.6 \text{ cm} \times \frac{57^\circ}{180^\circ} \times 8.6\text{ cm} \]

\[ \text{Area} = \frac{1}{2} \times 8.6 \text{ cm} \times 0.75 \times 8.6\text{ cm} \]

\[ \text{Area} = 28.83 \text{ cm}^2 \]

\[ \text{Area} \approx 28.8 \text{ cm}^2 \]
The diagram shows one disc with centre $A$ and radius 4 cm and another disc with centre $B$ and radius $x$ cm.

The two discs fit exactly into a rectangular box 10 cm long and 9 cm wide.

The two discs touch at $P$.

$APB$ is a straight line.

(a) Use Pythagoras’ Theorem to show that $x^2 - 30x + 45 = 0$

(b) Find the value of $x$.

Give your value correct to 3 significant figures.