London Examinations IGCSE
Mathematics
Paper 3H
Higher Tier
Monday 7 June 2010 – Afternoon
Time: 2 hours

Materials required for examination
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator.
Tracing paper may be used.

Items included with question papers
Nil

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
Without sufficient working, correct answers may be awarded no marks.
You must NOT write on the formulae page. Anything you write on the formulae page will gain NO credit.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 21 questions in this question paper. The total mark for this paper is 100.
There are 20 pages in this question paper. Any blank pages are indicated.
You may use a calculator.

Advice to Candidates
Write your answers neatly and in good English.
IGCSE MATHEMATICS 4400
FORMULA SHEET – HIGHER TIER

Pythagoras’ Theorem

\[ a^2 + b^2 = c^2 \]

\[ \text{adj} = \text{hyp} \times \cos \theta \]
\[ \text{opp} = \text{hyp} \times \sin \theta \]
\[ \text{opp} = \text{adj} \times \tan \theta \]

\[ \sin \theta = \frac{\text{opp}}{\text{hyp}} \]
\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} \]
\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]

Volume of cone = \( \frac{1}{3} \pi r^2 h \)
Curved surface area of cone = \( \pi rl \)
Volume of sphere = \( \frac{4}{3} \pi r^3 \)
Surface area of sphere = \( 4\pi r^2 \)

In any triangle \( ABC \)

Sine rule:
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Cosine rule:
\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Area of triangle = \( \frac{1}{2} ab \sin C \)

Volume of prism = area of cross section \( \times \) length

Circumference of circle = \( 2\pi r \)
Area of circle = \( \pi r^2 \)

Volume of cylinder = \( \pi r^2 h \)
Curved surface area of cylinder = \( 2\pi rh \)

Area of a trapezium = \( \frac{1}{2} (a + b)h \)

The Quadratic Equation
The solutions of \( ax^2 + bx + c = 0 \),
where \( a \neq 0 \), are given by
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY ONE questions.
Write your answers in the spaces provided.
You must write down all stages in your working.

1. Here are the ingredients needed to make Apple Fool for 6 people.

<table>
<thead>
<tr>
<th>Apple Fool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ingredients for 6 people</td>
</tr>
<tr>
<td>900 g cooking apples</td>
</tr>
<tr>
<td>100 g sugar</td>
</tr>
<tr>
<td>300 ml double cream</td>
</tr>
</tbody>
</table>

(a) Work out the amount of sugar needed to make Apple Fool for 15 people.

................. g  
(2)

(b) Work out the amount of cooking apples needed to make Apple Fool for 5 people.

................. g  
(2)  Q1

(Total 4 marks)
2. (a) (i) Find the value of $x$.

$x = ....................$

(ii) Give a reason for your answer.

................................................................................................................................

(2)

(b) (i) Find the value of $y$.

$y = ....................$

(ii) Give a reason for your answer.

................................................................................................................................

(2)

(Total 4 marks)

3. Three numbers $a$, $b$ and $c$ have a median of 4 and a range of 7

(a) Find the median of the three numbers $a + 2$, $b + 2$ and $c + 2$

..............................................

(1)

(b) Find the range of the three numbers $a + 2$, $b + 2$ and $c + 2$

..............................................

(1)

(Total 2 marks)
4. (a) Multiply out $5(n + 6)$

\[ \text{\hspace{1cm}} \]

(b) Simplify $y \times y \times y \times y \times y \times y$

\[ \text{\hspace{1cm}} \]

(c) Solve $4(x - 2) = 3$

\[ x = \text{\hspace{1cm}} \]

5. (a) $\frac{3}{10}$ of the members of a tennis club are men.

$\frac{5}{6}$ of these men are right-handed.

Work out the fraction of the members of the tennis club who are right-handed men.

\[ \text{\hspace{1cm}} \]

(b) $\frac{7}{12}$ of the members of a badminton club are women.

$\frac{3}{8}$ of the members of the badminton club wear glasses.

Work out the smallest possible number of members of the badminton club.

\[ \text{\hspace{1cm}} \]
6. The table shows information about the volume of water, in m$^3$, used by each of 80 families in one year.

<table>
<thead>
<tr>
<th>Volume of water ($V$ m$^3$)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; V \leq 100$</td>
<td>2</td>
</tr>
<tr>
<td>$100 &lt; V \leq 200$</td>
<td>4</td>
</tr>
<tr>
<td>$200 &lt; V \leq 300$</td>
<td>6</td>
</tr>
<tr>
<td>$300 &lt; V \leq 400$</td>
<td>18</td>
</tr>
<tr>
<td>$400 &lt; V \leq 500$</td>
<td>44</td>
</tr>
<tr>
<td>$500 &lt; V \leq 600$</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Write down the modal class.

.................................................................

(1)

(b) Work out an estimate for the mean volume of water used by the 80 families.

........... m$^3$

(4)
(c) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Volume of water (V m³)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; V ≤ 100</td>
<td></td>
</tr>
<tr>
<td>0 &lt; V ≤ 200</td>
<td></td>
</tr>
<tr>
<td>0 &lt; V ≤ 300</td>
<td></td>
</tr>
<tr>
<td>0 &lt; V ≤ 400</td>
<td></td>
</tr>
<tr>
<td>0 &lt; V ≤ 500</td>
<td></td>
</tr>
<tr>
<td>0 &lt; V ≤ 600</td>
<td></td>
</tr>
</tbody>
</table>

(d) On the grid, draw a cumulative frequency graph for your table.

(e) Use your graph to find an estimate for the median volume of water used by the 80 families.

............... m³

(Total 10 marks)
7. Work out the value of $x$.
   Give your answer correct to 3 significant figures.

   $x = ...............$

8. Jade has tax deducted from her income at the rate of 24%.
   Last month, after tax had been deducted, $1786 of her income remained.
   Calculate her income last month before the tax was deducted.

   $\$ ..............$

Diagram NOT accurately drawn
9.

(a) Describe fully the single transformation which maps triangle $P$ onto triangle $Q$.
............................................................................................................................................... (2)

(b) Rotate triangle $Q$ through $90^\circ$ anti-clockwise about the point $(-1,1)$.
Label the new triangle $R$.
............................................................................................................................................... (2)

(c) Describe fully the single transformation which maps triangle $P$ onto triangle $R$.
............................................................................................................................................... (2)

(Total 6 marks)
10. (a) An inequality is shown on the number line.

Write down this inequality.

................................................................. (2)

(b) (i) Solve the inequality $2x + 9 > 1$

................................................................. (4)

(ii) $n$ is a **negative** integer.

Write down all the values of $n$ which satisfy $2n + 9 > 1$
11. The diagram shows a fish bowl.
The water surface is a circle with a diameter of 16 cm.

(a) Work out the area of a circle with a diameter of 16 cm.
   Give your answer correct to 3 significant figures.

\[ \text{Area} = \pi \times \left( \frac{16}{2} \right)^2 \]

\[ \text{Area} = \frac{256}{4} \pi \]

\[ \text{Area} = 64 \pi \]

\[ \text{Area} \approx 201.06 \text{ cm}^2 \]

(b) The volume of water, \( V \) cm\(^3\), in the fish bowl may be found using the formula

\[ V = \frac{1}{3} \pi h (3x^2 + 3y^2 + h^2) \]

Find the value of \( V \) when \( h = 16.4 \) cm,
\( x = 6.5 \) cm,
and \( y = 8 \) cm.

\[ V = \frac{1}{3} \pi \times 16.4 \times (3 \times 6.5^2 + 3 \times 8^2 + 16.4^2) \]

\[ V \approx 3949.7 \text{ cm}^3 \]

\( V \approx 3950 \text{ cm}^3 \) (2 marks)

(Total 4 marks)
12. (a) Complete the table of values for $y = x^3 - 12x + 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$-7$</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

(b) On the grid, draw the graph of $y = x^3 - 12x + 2$ for values of $x$ from $-3$ to $4$
(c) For the curve with equation \( y = x^3 - 12x + 2 \)

(i) find \( \frac{dy}{dx} \) ..........................................

(ii) find the gradient of the curve at the point where \( x = 5 \) ..........................................


13. \( \quad \)

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Triangle $ABC$ is right-angled at $B$.
$AB = 20$ cm, correct to 1 significant figure.
$BC = 8.3$ cm, correct to 2 significant figures.

(a) Write down the lower bound for the length of
(i) $AB$,$\quad$ .................. cm
(ii) $BC$,$\quad$ .................. cm (2)

(b) Calculate the lower bound for the area of triangle $ABC$.

.................. cm$^2$ (2)

(c) Calculate the lower bound for the value of $\tan x^\circ$.

.................. (3)

(Total 7 marks)
15. The light intensity, \( E \), at a surface is inversely proportional to the square of the distance, \( r \), of the surface from the light source.

\[ E = \frac{k}{r^2} \]

\( E = 4 \) when \( r = 50 \)

(a) Express \( E \) in terms of \( r \).

\[ E = \frac{400}{r^2} \]  

(3)

(b) Calculate the value of \( E \) when \( r = 20 \)

\[ E = \frac{400}{20^2} = \frac{400}{400} = 1 \]  

(1)

(c) Calculate the value of \( r \) when \( E = 1600 \)

\[ r = \sqrt{\frac{400}{1600}} \]  

(2)

16. Show that \( (3 - \sqrt{5})^2 = 14 - 6\sqrt{5} \)
Two prisms, P and Q, are similar.
The cross-section of prism P is a triangle with a base of length 12 cm.
The cross-section of prism Q is a triangle with a base of length 18 cm.
The total surface area of prism P is 544 cm$^2$.

Calculate the total surface area of prism Q.

\[ \text{Total surface area of Q} \]

\[ \text{cm}^2 \]

(Total 3 marks)

18. Simplify fully

\[ \frac{x^2 + 6x}{x^2 - 36} \]

\[ \frac{\text{..................}}{\text{..................}} \]

(Total 3 marks)
19.

Ashok has six coins in his pocket.
He has one 5 cent coin, two 10 cent coins and three 20 cent coins.
He takes at random a coin from his pocket.
He records its value and puts the coin back into his pocket.
He then takes at random a second coin from his pocket and records its value.

(a) Calculate the probability that he takes two 20 cent coins.

........................ (2)

(b) Calculate the probability that the second coin he takes has a higher value than the first coin he takes.

........................ (3)

(Total 5 marks)
$A$, $B$ and $C$ are points on horizontal ground.
$C$ is due West of $B$.
$A$ is due South of $B$ and $AB = 40$ m.
There is a vertical flagpole at $B$.
From $A$, the angle of elevation of the top of the flagpole is $13^\circ$.
From $C$, the angle of elevation of the top of the flagpole is $19^\circ$.

Calculate the distance $AC$.
Give your answer correct to 3 significant figures.
21. Solve the simultaneous equations

\[ \begin{align*}
  y &= 2x^2 \\
  y &= 3x + 14
\end{align*} \]