London Examinations IGCSE
Mathematics
Paper 3H
Higher Tier
Thursday 5 November 2009 – Morning
Time: 2 hours

Materials required for examination
Ruler graduated in centimetres and
millimetres, protractor, compasses,
pen, HB pencil, eraser, calculator.
Tracing paper may be used.

Items included with question papers
Nil

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
Without sufficient working, correct answers may be awarded no marks.
You must NOT write on the formulae page. Anything you write on the formulae page will gain
NO credit.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 25 questions in this question paper. The total mark for this paper is 100.
There are 24 pages in this question paper. Any blank pages are indicated.
You may use a calculator.

Advice to Candidates
Write your answers neatly and in good English.
**Pythagoras’ Theorem**

\[ a^2 + b^2 = c^2 \]

- \( \text{adj} = \text{hyp} \times \cos \theta \)
- \( \text{opp} = \text{hyp} \times \sin \theta \)
- \( \text{opp} = \text{adj} \times \tan \theta \)

or

\[ \sin \theta = \frac{\text{opp}}{\text{hyp}} \]
\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} \]
\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]

**Volume of cone** = \( \frac{1}{3} \pi r^2 h \)

**Volume of sphere** = \( \frac{4}{3} \pi r^3 \)

**Curved surface area of cone** = \( \pi rl \)

**Surface area of sphere** = \( 4\pi r^2 \)

**Circumference of circle** = \( 2\pi r \)

**Area of circle** = \( \pi r^2 \)

**Volume of cylinder** = \( \pi r^2 h \)

**Curved surface area of cylinder** = \( 2\pi rh \)

**Area of a trapezium** = \( \frac{1}{2}(a + b)h \)

**In any triangle** \( ABC \)

- **Sine rule:** \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)
- **Cosine rule:** \( a^2 = b^2 + c^2 - 2bc \cos A \)
- **Area of triangle** = \( \frac{1}{2} ab \sin C \)

**The Quadratic Equation**

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. Show that \[ \frac{2}{3} + \frac{1}{5} = \frac{13}{15} \]

2. Solve \[ 8y - 9 = 5y + 3 \]

\[ y = \ldots \]

Q1 (Total 2 marks)

Q2 (Total 3 marks)
3. (a) The diagram shows a regular octagon, with centre $O$.

![Diagram of a regular octagon with centre O and angle $x$]

Work out the value of $x$.

$x = \ldots .....

(3)

(b) A regular polygon has an exterior angle of $30^\circ$.
Work out the number of sides of the polygon.

$\ldots .....

(2)

(Total 5 marks)
4. In a survey of 36 families, the number of people in each family was recorded. The table shows the results.

<table>
<thead>
<tr>
<th>Number of people in the family</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

Work out the mean number of people in these 36 families.
5. Cups cost \( x \) dollars each.
   Mugs cost \((x + 2)\) dollars each.

(a) Write down an expression, in terms of \( x \), for the total cost of 12 cups and 6 mugs.

................................................................. dollars  

(2)

(b) The total cost of 12 cups and 6 mugs is 57 dollars.
   Work out the cost of 1 cup.

........................................... dollars  

(2)

(Total 4 marks)
6. (a) \[ S = \{1, 3, 5, 7\} \]
\[ T = \{2, 3, 7, 11\} \]

How many members are there in \( S \cup T \)?

...................................................

(1)

(b) \[ U = \{3, 4, 5\} \]
\[ U \cup V = \{1, 2, 3, 4, 5\} \]

The set \( V \) has as few members as possible.
List the members of the set \( V \).

...................................................

(1)

(c) \[ A = \{\text{Cats}\} \]
\[ B = \{\text{Black animals}\} \]

Describe the members of \( A \cap B \).

...................................................

(1)

Q6

(Total 3 marks)
7. (a) Calculate the circumference of a circle of radius 30 cm.
   Give your answer correct to 3 significant figures.

\[ .............................. \text{ cm} \]
(2)

(b) The diagram shows a circle with radius 2.1 cm inside a square.
   The circle touches the sides of the square.

Diagram NOT accurately drawn

Work out the shaded area.
Give your answer correct to 3 significant figures.

\[ .............................. \text{ cm}^2 \]
(4)

(Total 6 marks)
8. James throws a biased dice once.
The table shows all the possible scores and their probabilities.

<table>
<thead>
<tr>
<th>Score</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Find the probability that the score is more than 3
9. (a) Expand and simplify fully \(2(w - 3) + 3(w + 5)\)
\[\text{............................................... (2)}\]

(b) Solve the equation \(\frac{x+5}{3} = 9\)
\[x = \text{............................................... (2)}\]

(c) Solve the inequality \(5y + 7 < 13\)
\[\text{............................................... (2)}\]

(Total 6 marks)
10. The diagram shows a prism.
   The cross section of the prism is a right-angled triangle.
   The lengths of the sides of the triangle are 8 cm, 15 cm and 17 cm.
   The length of the prism is 20 cm.
   Work out the total surface area of the prism.

\[ \text{Total surface area} = \text{(2 x Area of triangle) + (4 x Perimeter of triangle)} \]

\[ \text{Area of triangle} = \frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2 \]

\[ \text{Perimeter of triangle} = 8 + 15 + 17 = 40 \text{ cm} \]

\[ \text{Total surface area} = (2 \times 60) + (4 \times 40) = 120 + 160 = 280 \text{ cm}^2 \]

11. Make \( a \) the subject of \( P = \sqrt{ab} \)

\[ a = \frac{P^2}{b} \]
12. (a) Calculate the value of $x$.
Give your answer correct to 3 significant figures.

\[ x = \ldots \] (3)

(b) Calculate the value of $y$.
Give your answer correct to 3 significant figures.

\[ y = \ldots \] (3)

(Total 6 marks)
13. The table shows the area, in km$^2$, of some countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Area (km$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>$2.4 \times 10^6$</td>
</tr>
<tr>
<td>Botswana</td>
<td>$6.0 \times 10^5$</td>
</tr>
<tr>
<td>Equatorial Guinea</td>
<td>$2.8 \times 10^4$</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>Malawi</td>
<td>$1.2 \times 10^5$</td>
</tr>
</tbody>
</table>

(a) Which of these countries has the largest area?

...................................................

(1)

(b) How many times greater is the area of Ethiopia than the area of Malawi?

.............................................

(1)

(c) Work out the total area of all five countries.

Give your answer in standard form.

............................................. km$^2$

(2)

(Q13)

(Total 4 marks)
14. Solve the simultaneous equations

\[ 2x - 3y = 3 \]
\[ 3x + 6y = 1 \]

\[ x = \ldots \]
\[ y = \ldots \]

15. Jothi bought a car.
    Later, Jothi sold the car for £2125
    He made a loss of 15%.
    Work out the original price of the car.

£ \ldots

(Total 3 marks)
16. The cumulative frequency diagram shows information about the heights, in centimetres, of 200 plants.

(a) Find an estimate for the median height.

................................... cm

(2)

(b) Work out an estimate for the number of plants whose heights are greater than 80 cm.

...................................  

(2)

(Total 4 marks)
17. (a) Factorise $x^2 - y^2$

........................................................................................................(1)

(b) Factorise completely $(c + d)^2 - d^2$

........................................................................................................(2)

(c) Factorise $2w^2 + w - 3$

........................................................................................................(2)

(Total 5 marks)
18. In the diagram, a sector of a circle of radius 12 cm is shaded. The area of the sector is $112\pi \text{cm}^2$. Calculate the value of $x$. 

\[ x = \text{.............................} \]

Diagram NOT accurately drawn

(Total 4 marks)
19. (a) Simplify \( \frac{x^2}{x^2 - 2x} \)

(b) Simplify \( \frac{2}{2x-1} - \frac{1}{x+1} \)
Each time Jeni plays a computer game the probability that she will win is \( \frac{2}{3} \).

Jeni plays the computer game 3 times.

Calculate the probability that Jeni will win

(a) all 3 games,

(b) exactly 2 out of the 3 games.
21. $t$ is proportional to the square root of $d$.

$t = 12$ when $d = 4$

(a) Find a formula for $t$ in terms of $d$.

....................................

(b) Calculate the value of $t$ when $d = 9$

$t = .........................$

(Total 5 marks)
22. The diagram shows the positions of two ships, $A$ and $B$, and a lighthouse $L$.

Ship $A$ is 5 km from $L$ on a bearing of $070^\circ$ from $L$.
Ship $B$ is 3 km from $L$ on a bearing of $210^\circ$ from $L$.
Calculate the distance between ship $A$ and ship $B$.
Give your answer correct to 3 significant figures.

............................... km

(Total 3 marks)
23. In a race, Paula runs 25 laps of a track. Each lap of the track is 400 m, correct to the nearest metre. Paula’s average speed is 5.0 m/s, correct to one decimal place.

Calculate the upper bound for the time that Paula takes to run the race. Give your answer in minutes and seconds, correct to the nearest second.
24. $f(x) = x^2$
   $g(x) = x - 3$

(a) (i) Find $gf(x)$.....................................

(ii) Find $g^{-1}(x)$.....................................

(b) Solve the equation $gf(x) = g^{-1}(x)$

.........................

(2)

.........................

(3)

(Total 5 marks)
25. (a) \((\sqrt{a})^7 = k\sqrt{a}\), where \(k = a^n\)

Find the value of \(n\).

\[ n = \ldots \] (2)

(b) Express \(\frac{1}{2\sqrt{2}}\) as a power of 2

\[ \ldots \] (2)