London Examinations IGCSE
Mathematics
Paper 3H
Higher Tier
Monday 18 May 2009 – Afternoon
Time: 2 hours

Materials required for examination
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator.
Tracing paper may be used.

Items included with question papers
Nil

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
Without sufficient working, correct answers may be awarded no marks.
You must NOT write on the formulae page. Anything you write on the formulae page will gain NO credit.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 21 questions in this question paper. The total mark for this paper is 100.
There are 24 pages in this question paper. Any blank pages are indicated.
You may use a calculator.

Advice to Candidates
Write your answers neatly and in good English.
IGCSE MATHEMATICS 4400
FORMULA SHEET – HIGHER TIER

Pythagoras’ Theorem

\[ a^2 + b^2 = c^2 \]

Area of a triangle = \( \frac{1}{2} \) \( \text{base} \times \text{height} \)

\[ \text{Volume of prism} = \text{area of cross section} \times \text{length} \]

\[ \text{Surface area of sphere} = 4\pi r^2 \]

\[ \text{Volume of cone} = \frac{1}{3} \pi r^2 h \]

\[ \text{Curved surface area of cone} = \pi rl \]

\[ \text{Volume of sphere} = \frac{4}{3} \pi r^3 \]

\[ \text{Volume of cylinder} = \pi r^2 h \]

\[ \text{Curved surface area of cylinder} = 2\pi rh \]

In any triangle \( ABC \)

Sine rule: \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine rule: \( a^2 = b^2 + c^2 - 2bc \cos A \)

Area of triangle = \( \frac{1}{2} \) \( \text{base} \times \text{height} \)

The Quadratic Equation

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY ONE questions.
Write your answers in the spaces provided.
You must write down all stages in your working.

1. Last year in Mathstown High School, the ratio of the number of candidates for IGCSE mathematics to the number of candidates for IGCSE biology was 5 : 2
   The number of candidates for IGCSE mathematics was 80
   
   (a) Work out the number of candidates for IGCSE biology.

   ..................  
   (2)

   The 80 mathematics candidates were divided between Foundation and Higher in the ratio 1 : 3
   
   (b) Work out the number of Foundation candidates.

   ..................  
   (2)

   (Total 4 marks)

2. Omar travelled from Nairobi to Mombasa by train.
   The journey took 13 hours 15 minutes.
   The average speed was 40 km/h.
   
   Work out the distance from Nairobi to Mombasa.

   .................. km  
   (Total 3 marks)

(Turn over)
3. On the grid, enlarge triangle $T$ with a scale factor of $2\frac{1}{2}$ and centre $(0, 0)$.

4. A bag contains 10 coloured beads. Ella is going to take at random a bead from the bag. She says, “The probability that I will take a red bead is 0.35”

   Explain why Ella is wrong.
   You must show working to justify your answer.

   ..............................................................................................................................................
   ..............................................................................................................................................
   ..............................................................................................................................................
5. (a) Factorise $p^2 + 7p$

(b) Solve $4 - 5x = 2$

(c) Simplify $t^3 \times t^6$

(d) Expand and simplify $3(4y + 5) - 5(2y + 3)$

(Q5 Total 8 marks)
6. Brett’s weekly pay is $760.
   He spends $266 on rent.
   (a) Express $266 as a percentage of $760.

   $\ldots\ldots\ldots\%$

   \(\ldots\ldots\ldots\%\) (2)

   Kazia spends $204 a week on rent.
   $204$ is 30% of her weekly pay.
   (b) Work out her weekly pay.

   $\ldots\ldots\ldots$

   $\ldots\ldots\ldots$ (2)

   (Total 4 marks)
Work out the value of $x$.
Give your answer correct to 1 decimal place.

\[ x = \ldots \ldots \ldots \]

(Total 3 marks)
8. \[ X = \{ \text{positive whole numbers} \} \]
\[ A = \{ \text{factors of 27} \} \]
\[ B = \{ \text{factors of 9} \} \]
\[ C = \{ \text{first four even numbers} \} \]

(a) List the members of \( A \cup B \).

..................................................................

(2)

(b) (i) Is it true that \( A \cap C = \emptyset \)?

Tick (✔) the appropriate box.

Yes ☐ No ☐

(ii) Explain your answer.

................................................................................................................................

................................................................................................................................

(1)

(c) Complete the Venn Diagram to show the relationship between the sets \( A, B \) and \( C \).

.................................................................................................................................

.................................................................................................................................

(2)

(Total 5 marks)
A is a point on a circle with centre $O$ and radius 4.7 cm. 
$AB$ is the tangent to the circle at $A$. 
$AB = 5.9$ cm. 
$OB$ intersects the circle at $C$. 

Calculate the length of $BC$. 
Give your answer correct to 3 significant figures.
The table shows information about the distances walked in a week by 40 people.

<table>
<thead>
<tr>
<th>Distance (d km)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; d \leq 20$</td>
<td>8</td>
</tr>
<tr>
<td>$20 &lt; d \leq 40$</td>
<td>24</td>
</tr>
<tr>
<td>$40 &lt; d \leq 60$</td>
<td>5</td>
</tr>
<tr>
<td>$60 &lt; d \leq 80$</td>
<td>2</td>
</tr>
<tr>
<td>$80 &lt; d \leq 100$</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Work out an estimate for the mean distance walked in a week by the 40 people.

\[
\text{Mean distance} = \frac{\sum (d \times f)}{\sum f}
\]

\[
\text{Mean distance} = \frac{\text{Total distance}}{\text{Total frequency}}
\]

\[
\text{Mean distance} = \frac{(8 \times 10) + (24 \times 30) + (5 \times 50) + (2 \times 70) + (1 \times 90)}{40}
\]

\[
\text{Mean distance} = \frac{80 + 720 + 250 + 140 + 90}{40}
\]

\[
\text{Mean distance} = \frac{1280}{40}
\]

\[
\text{Mean distance} = 32 \text{ km}
\]
The information in the table was used to draw the cumulative frequency graph.

(b) Find an estimate for the number of people who walked less than 25 km.

............... km

(2)

(c) Find an estimate for the interquartile range of the distances walked by the 40 people.

............... km

(2)

(Total 8 marks)
11. (a) Solve the simultaneous equations

\[ 2x - 3y = 9 \]
\[ 5x + 4y = 11 \]

\[ x = \ldots \ldots \ldots \ldots \]
\[ y = \ldots \ldots \ldots \ldots \]

(4)

(b) Write down the coordinates of the point of intersection of the two lines whose equations are \( 2x - 3y = 9 \) and \( 5x + 4y = 11 \)

\[ (\ldots \ldots \ldots \ldots , \ldots \ldots \ldots \ldots ) \]

(1)

(Total 5 marks)
12. 1 astronomical unit = 150 million kilometres.

(a) Write the number 150 million in standard form.

.........................................................

(2)

The distance from Venus to the Sun is 108 million kilometres.

(b) Express 108 million kilometres in astronomical units.
    Give your answer in standard form.

.................. astronomical units

(2)

(Total 4 marks)
13. Here is a cuboid with length $L$, width $W$ and height $H$.

The total surface area, $A$, of the cuboid is given by the formula

$$A = 2(LW + HW + HL)$$

(a) $A = 70$  $W = 4$  $H = 2$
Work out the value of $L$.

$$L = \ldots$$ (3)

(b) Make $W$ the subject of the formula $A = 2(LW + WH + HL)$

$$W = \ldots$$ (4)
A, B, C and D are points on a circle. 
$ABE$ and $DCE$ are straight lines. 
$AT$ is a tangent to the circle. 
$DCE$ is parallel to $AT$. 
Angle $EAT = 47^\circ$. Angle $BAD = 56^\circ$.

(a) (i) Find the size of angle $AED$. 
..................... ° 

(ii) Give a reason for your answer.
....................................................................................................................................................................................... 
....................................................................................................................................................................................... 

(b) Find the size of angle $BCD$. 
..................... ° 

(c) (i) Find the size of angle $ADB$. 
..................... ° 

(ii) Give a reason for your answer.
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(Total 5 marks)
15. The diagram shows part of the graph of \( y = f(x) \) and part of the graph of \( y = g(x) \).

(a) Find \( f(3) \).

(b) Solve \( f(x) = g(x) \).
   Give your answers correct to 1 decimal place.

(c) Find \( fg(1) \).
(d) Find an estimate for the gradient of the graph of \( y = f(x) \) at the point (1, 16).

\[
\quad
\]

A solid cone, \( P \), has a base radius of 4 cm and a slant height of 9 cm.

(a) Calculate the total surface area of the cone.
Give your answer correct to 3 significant figures.

\[
\text{cm}^2
\]

Another solid cone, \( Q \), is similar to \( P \).
The base radius of \( Q \) is 6 cm.
The volume of \( Q \) is \( k \) times the volume of \( P \).

(b) Calculate the value of \( k \).

\[
k = \quad
\]

(Total 4 marks)
17. Here are five counters.
    Each counter has a number on it.

\[ \begin{array}{c}
1 & 3 & 3 & 3 & 5 \\
\end{array} \]

Layla puts the five counters in a bag.
She takes two counters at random from the bag without replacement.

Calculate the probability that

(i) both counters will have the number 3 on them,

(ii) the sum of the numbers on the two counters will be 6

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(Total 5 marks)
18. Simplify fully $\frac{5x^2 + 14x - 3}{50x^2 - 2}$
The diagram shows a sector $OAPB$ of a circle, centre $O$. 
$AB$ is a chord of the circle. 
The radius of the circle is 6 cm. 
Angle $AOB = 78^\circ$. 

Calculate the perimeter of the shaded segment $APB$. 
Give your answer correct to 3 significant figures.
20. Correct to 2 significant figures, the area of a square is 230 cm$^2$.

Calculate the lower bound for the perimeter of the square.

\[ \text{\underline{cm}} \]

Q20

(Total 3 marks)
The diagram shows the length, in centimetres, of each side of triangle $ABC$. Angle $BAC = 60^\circ$. Find the value of $x$. 

$x = \ldots$