Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided — **there may be more space than you need**.
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain **NO** credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets — **use this as a guide as to how much time to spend on each question**.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
**FORMULAE SHEET – HIGHER TIER**

**Pythagoras’ Theorem**

\[ a^2 + b^2 = c^2 \]

Volume of cone = \( \frac{1}{3} \pi r^2 h \)

Curved surface area of cone = \( \pi rl \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)

Surface area of sphere = \( 4\pi r^2 \)

**Theorem**

\[ a^2 + b^2 = c^2 \]

adj = hyp \( \times \) cos \( \theta \)

opp = hyp \( \times \) sin \( \theta \)

opp = adj \( \times \) tan \( \theta \)

or \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \)

\( \cos \theta = \frac{\text{adj}}{\text{hyp}} \)

\( \tan \theta = \frac{\text{opp}}{\text{adj}} \)

**In any triangle ABC**

Sine rule: \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine rule: \( a^2 = b^2 + c^2 - 2bc \cos A \)

Area of triangle = \( \frac{1}{2} ab \sin C \)

**Volume of prism** = area of cross section \( \times \) length

**Area of circle** = \( \pi r^2 \)

Circumference of circle = \( 2\pi r \)

Volume of cylinder = \( \pi r^2 h \)

Curved surface area of cylinder = \( 2\pi rh \)

**Area of a trapezium** = \( \frac{1}{2}(a + b)h \)

**The Quadratic Equation**

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. Work out the value of \( \frac{6.6 \times 1.2}{4.4 - 2.75} \)

(Total for Question 1 is 2 marks)

2. A group of students take a test.
   The group consists of 12 boys and 8 girls.
   The mean mark for the boys is 18
   The mean mark for the girls is 16.5
   Calculate the mean mark for the whole group.

(Total for Question 2 is 4 marks)
Bhavik left his home at 12:00 to cycle to Sam’s house. On the way Bhavik stopped for a rest, and then continued his journey. The distance-time graph shows his journey.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance from home (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>13:00</td>
<td>15</td>
</tr>
<tr>
<td>14:00</td>
<td>25</td>
</tr>
<tr>
<td>15:00</td>
<td>30</td>
</tr>
<tr>
<td>16:00</td>
<td>35</td>
</tr>
<tr>
<td>17:00</td>
<td>40</td>
</tr>
<tr>
<td>18:00</td>
<td>45</td>
</tr>
</tbody>
</table>

(a) (i) For how many minutes did Bhavik stop for a rest?

.............................................................. minutes

(ii) After his rest, how many more kilometres did Bhavik cycle to Sam’s house?

.............................................................. km

(b) Bhavik stayed at Sam’s house for 2 hours. He then cycled back to his home. He arrived home at 17:15.

Show all this information on the graph.

(c) Write down the times at which Bhavik was 24 kilometres from his home.

..............................................................

..............................................................

..............................................................
(d) Work out the average speed, in kilometres per hour, of Bhavik’s journey from Sam’s house back to his home.

Give your answer correct to 1 decimal place.

\[ \text{\underline{\hspace{2cm}}} \text{ km/h} \]

(3)

(Total for Question 3 is 9 marks)

4

(a) Describe fully the single transformation that maps shape P onto shape Q.

(b) On the grid, rotate shape P 90° clockwise about the point (2, 0).
Label the new shape R.

(Total for Question 4 is 4 marks)
5 (a) Show that \( \frac{4}{5} \div \frac{7}{15} = 1 \frac{5}{7} \) (2)

(b) Show that \( 5 \frac{1}{4} - 1 \frac{2}{3} = 3 \frac{7}{12} \) (3)

(Total for Question 5 is 5 marks)

6

Diagram NOT accurately drawn

Calculate the length of \( MN \). Give your answer correct to 3 significant figures.

\[ \text{........................................ cm} \]

(Total for Question 6 is 3 marks)
7. Showing clear algebraic working, solve the simultaneous equations

\[ 3a + 2b = 1 \]
\[ a + 2b = 5 \]

\[ a = \ldots \]
\[ b = \ldots \]

(Total for Question 7 is 3 marks)

8. Express 300 as a product of its prime factors.

\ldots 

(Total for Question 8 is 3 marks)
9. The table shows information about the snowfall in Ottawa in January one year.

<table>
<thead>
<tr>
<th>Snowfall (s cm)</th>
<th>Number of days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ s &lt; 2</td>
<td>19</td>
</tr>
<tr>
<td>2 ≤ s &lt; 4</td>
<td>8</td>
</tr>
<tr>
<td>4 ≤ s &lt; 6</td>
<td>3</td>
</tr>
<tr>
<td>6 ≤ s &lt; 8</td>
<td>0</td>
</tr>
<tr>
<td>8 ≤ s &lt; 10</td>
<td>1</td>
</tr>
</tbody>
</table>

Work out an estimate for the total snowfall in January.

.............................................................. cm

(Total for Question 9 is 3 marks)

10. (a) Expand and simplify

(i) \(5(2x + 1) - 3(3x - 1)\)

..............................................................

(ii) \((y + 5)(y - 7)\)

..............................................................

(4)

(b) Make \(r\) the subject of the formula \(V = \pi r^2 h\) where \(r\) is positive.

..............................................................

\(r = \) ..............................................................

(2)

(Total for Question 10 is 6 marks)
11 The mass of the Space Shuttle is $7.8 \times 10^4$ kilograms.

(a) Write $7.8 \times 10^4$ as an ordinary number.

..............................................................

(1)

The Space Shuttle docks with the International Space Station.
The mass of the International Space Station is $4.62 \times 10^5$ kilograms.

(b) Calculate the total mass of the Space Shuttle and the International Space Station.
Give your answer in standard form.

.............................................................. kg

(2)

(Total for Question 11 is 3 marks)

12 $A$, $B$ and $C$ are three sets.

$A \cap B = \emptyset$ and $C \subseteq A$

(a) Complete the Venn diagram to show the sets $B$ and $C$

..............................................................

(2)

(b) On the Venn diagram, shade the region that represents $A \cap C'$

..............................................................

(1)

(Total for Question 12 is 3 marks)
13 (a) Complete the table of values for \( y = x^3 - 3x - 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) On the grid, draw the graph of \( y = x^3 - 3x - 1 \) for \(-2 \leq x \leq 3\)

(c) By drawing a suitable straight line on the grid, find an estimate for the solution of the equation \( x^3 - 3x - 6 = 0 \)

Give your answer correct to 1 decimal place.

..............................................................
(d) For the curve with equation \( y = x^3 - 3x - 1 \)

(i) find \( \frac{dy}{dx} \) ..............................................................

(ii) find the gradient of the curve at the point where \( x = 4 \) ..............................................................

(4)

(Total for Question 13 is 10 marks)

14 There are 31 students in a class.

17 students study French.

15 students study Spanish.

6 students study neither French nor Spanish.

Using a Venn diagram, or otherwise, work out how many students study only one language.

(Total for Question 14 is 4 marks)
$A$, $B$, $C$ and $D$ are four points on a circle, centre $O$.
$AD$ is a diameter of the circle.
Angle $BAD = 58^\circ$

(a) Calculate the size of angle $ADB$.

(b) (i) Calculate the size of angle $BCD$.

(ii) Give a reason for your answer.
A, B, C and D are four points on a circle. 
ABX and DCX are straight lines.
AB = 7 cm, BX = 5 cm and CX = 6 cm.
Angle BXC = 28°

(a) Calculate the length of AC.
Give your answer correct to 3 significant figures.

.............................................................. cm

(b) Calculate the length of DC.

.............................................................. cm

(Total for Question 16 is 6 marks)
The histogram shows information about the times, \( t \) minutes, patients spent at a doctors’ surgery on one day. No patient spent more than 40 minutes at the surgery.

(a) Calculate the percentage of the patients who spent between 25 and 40 minutes at the surgery.

\[ \text{Percentage} \] %

(b) 16 patients spent between 10 and 15 minutes at the surgery.

Calculate the total number of patients at the surgery that day.

\[ \text{Total number of patients} \]
Bill and Jo play some games of table tennis.
The probability that Bill wins the first game is 0.7
When Bill wins a game, the probability that he wins the next game is 0.8
When Jo wins a game, the probability that she wins the next game is 0.5
The first person to win two games wins the match.

(a) Complete the probability tree diagram.

(b) Calculate the probability that Bill wins the match.

(Total for Question 18 is 6 marks)
19

\[ f(x) = 3x - 2 \]
\[ g(x) = \frac{10}{x + 2} \]

(a) Express the inverse function \( f^{-1} \) in the form \( f^{-1}(x) = \ldots \)

\[ f^{-1}(x) = \ldots \]

(b) Find \( gf(x) \)

Simplify your answer.

\[ gf(x) = \ldots \]

(Total for Question 19 is 4 marks)
20 Show that $(6 - \sqrt{8})^2 = 44 - 24\sqrt{2}$

Show each stage of your working clearly.

(Total for Question 20 is 3 marks)

21 Solve \( \frac{5}{x + 2} + \frac{9}{x - 2} = 2 \)

Show clear algebraic working.

(Total for Question 21 is 5 marks)
Two solid spheres, each of radius $r$ cm, fit exactly inside a hollow cylinder.
The radius of the cylinder is $r$ cm.
The height of the cylinder is equal to $4r$ cm.

The volume of the space inside the cylinder, not occupied by the spheres, is $\frac{125}{6} \pi$ cm$^3$.

Calculate the value of $r$.

Show your working clearly.

$$r = \ldots$$

(Total for Question 22 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS